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Utility Perturbation Operators in Bayesian Games: Structural Stability and Equilibrium Deformation

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Abstract

We introduce a class of parametric operators acting on the space of Bayesian games with continuous utility functions. Each operator induces a structured perturbation of agents' utilities while preserving the underlying informational primitives, strategy spaces, and Bayesian updating. This construction generates a family of utility-perturbed Bayesian games that can be interpreted as continuous deformations of classical incomplete-information games in the space of payoff functions. The contribution of the paper is purely mathematical. First, we formally define a utility perturbation operator and characterize the associated class of perturbed Bayesian games. Second, under standard compactness and continuity assumptions, we prove the existence of Nash equilibria for all admissible perturbations. Third, we show that the equilibrium correspondence of the perturbed games converges upper hemicontinuously to the classical Bayesian Nash equilibrium correspondence as the perturbation parameter vanishes. Under additional differentiability and strict concavity assumptions, we establish a structural stability result: in a neighborhood of the unperturbed game, equilibria are locally unique and depend smoothly on the perturbation parameter. The equilibrium mapping is continuous, locally Lipschitz, and differentiable, implying that utility perturbations generate a stable deformation of the classical equilibrium structure rather than a qualitative departure from it. Taken together, the results identify a new operator-based framework for studying equilibrium stability and sensitivity in Bayesian games. The analysis shows that parametric perturbations of utility functions define a mathematically well-posed deformation of classical game-theoretic equilibria, providing a foundation for further work on equilibrium equivalence, stability, and comparative statics in non-cooperative games.

Keywords: asymmetric information; Bayesian games; behavioral perturbations; equilibrium existence; comparative statics

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1. Introduction

Strategic interaction under incomplete information constitutes a central object of study in non-cooperative game theory. Classical Bayesian games formalize such interactions by specifying an informational structure—type spaces, priors, and Bayesian updating—together with payoff functions evaluated under expected utility maximization. This framework has

proved mathematically robust and analytically tractable, yielding fundamental results on equilibrium existence, refinement, and stability [1,2].

In the standard formulation, informational asymmetries enter exclusively through beliefs over types, while preferences are treated as fixed and objective. Consequently, equilibrium behavior can change only through variations in information or belief structures. From a mathematical perspective, this separation has the advantage of conceptual clarity but restricts the class of admissible perturbations to informational primitives, leaving the utility representation itself unchanged [3,4].

A growing body of theoretical work has explored extensions of classical games in which agents' payoff evaluation is systematically altered. Much of this literature originates in behavioral game theory and decision theory, documenting systematic deviations from expected utility maximization such as loss aversion, reference dependence, framing effects, and social interaction [5,6]. However, many existing approaches introduce such effects either descriptively or through application-specific functional adjustments, often blurring the distinction between changes in informational structure and changes in preferences. As a result, it remains an open mathematical question how perturbations of utility functions can be incorporated into Bayesian games while preserving equilibrium existence, continuity, and consistency with the classical framework [7].

The present paper addresses this question by introducing an operator-based approach to utility perturbations in Bayesian games. Rather than modifying beliefs, signals, or informational primitives, we define a class of parametric operators acting on the space of utility functions. Each operator induces a structured deformation of payoffs while leaving the underlying game—strategy spaces, type spaces, priors, and Bayesian updating—unchanged. This construction generates a family of utility-perturbed Bayesian games that can be interpreted as continuous deformations of classical incomplete-information games in the space of payoff functions.

From a mathematical standpoint, the contribution of the paper is twofold. First, we provide a formal definition of a utility perturbation operator and characterize the associated class of perturbed Bayesian games. Second, we analyze the equilibrium properties of this class. Under standard compactness, continuity, and quasi-concavity assumptions commonly imposed in non-cooperative game theory, we establish existence of Nash equilibria for all admissible perturbations using fixed-point arguments. Moreover, we show that the equilibrium correspondence of the perturbed games converges upper hemicontinuously to the classical Bayesian Nash equilibrium correspondence as the perturbation parameter vanishes, relying on standard results on continuity of correspondences.

Under additional differentiability and strict concavity assumptions, we obtain a structural stability result. In a neighborhood of the unperturbed game, equilibria are locally unique and depend smoothly on the perturbation parameter. The equilibrium mapping is continuous, locally Lipschitz, and differentiable, implying that utility perturbations generate a stable deformation of the classical equilibrium structure rather than a qualitative departure from it. This analysis connects the proposed framework with established results in variational analysis and parametric optimization.

This operator-based perspective clarifies in what precise sense the proposed framework extends Bayesian games. The informational structure remains fixed, while preferences are allowed to vary within a mathematically well-defined family. Equilibrium behavior is therefore affected through utility deformation rather than through changes in beliefs or signals. Deviations from classical equilibrium predictions arise as equilibrium-consistent outcomes within a unified analytical structure, rather than as violations of rationality or equilibrium logic.

The contribution of the paper is purely mathematical and model-driven. Any numerical illustrations included in the manuscript serve solely to visualize the equilibrium mechanisms implied by the theory and to verify internal coherence. They play no role in the formal analysis and do not support statistical inference or empirical validation.

2. Literature Review

The mathematical analysis of strategic interaction under asymmetric information originates in classical game theory, where informational heterogeneity is formalized through incomplete-information games and Bayesian equilibrium concepts. Seminal contributions established that unequal or incomplete information may generate equilibrium outcomes that deviate from competitive benchmarks through mechanisms such as adverse selection, signaling, and strategic mispricing, while preserving full rationality of agents and a fixed utility representation [2,8,9]. In this framework, inefficiencies arise exclusively from informational constraints rather than from distortions in preference evaluation.

Subsequent developments refined the mathematical structure of Bayesian games by characterizing equilibrium existence, continuity, and refinement under increasingly general conditions. The standard formulation assumes that agents maximize expected utility with respect to subjective beliefs satisfying Bayesian consistency, yielding a robust analytical framework in which deviations from classical predictions occur through changes in beliefs or information sets, while preferences remain exogenously fixed [1,3].

Parallel to these developments, a substantial body of work in decision theory has documented systematic departures from expected utility maximization, including loss aversion, reference dependence, and probability distortion [5]. From a mathematical perspective, however, much of this evidence remains external to the formal structure of non-cooperative games, as such deviations are typically documented empirically rather than embedded into equilibrium models.

Behavioral game theory seeks to reconcile these observations with equilibrium analysis by incorporating systematic deviations into strategic models while preserving equilibrium concepts. Early contributions introduce such effects either through modified payoff functions or through alternative response rules, allowing equilibrium behavior to reflect non-standard preference evaluation without abandoning analytical rigor [3,4]). Nevertheless, many of these approaches rely on application-specific functional forms or reduced-form representations, which may obscure the structure of equilibrium correspondences and complicate the analysis of continuity and stability.

More recent theoretical work has moved toward explicit parametric representations of utility distortions within utility-based models. In these formulations, loss sensitivity and related effects are introduced as additive or multiplicative components of utility, effectively defining a deformation of the classical expected-utility framework. A central admissibility requirement of such models is that classical Nash equilibria are recovered as limiting cases when perturbation parameters converge to zero, ensuring consistency with established results in non-cooperative game theory [10].

Related strands of the literature examine how framing or narrative emphasis may alter subjective payoff evaluation. While often motivated by empirical considerations, these mechanisms can be interpreted mathematically as structured perturbations of utility functions rather than as modifications of informational primitives. From this perspective, framing effects suggest the relevance of utility deformation as an analytical channel distinct from belief formation or signal structure [11–13].

Despite extensive contributions, relatively few studies provide a unified mathematical framework in which informational asymmetry and systematic utility perturbations are simultaneously represented as endogenous components of strategic interaction. In particular,

many models leave implicit the properties of the equilibrium correspondence induced by perturbations of preferences, limiting formal analysis of existence, continuity, stability, and sensitivity with respect to parameter changes [14].

The present study contributes to this literature by introducing a mathematically explicit framework in which utility perturbations are embedded directly into agents' payoff functions, while informational asymmetry is treated in the standard Harsanyi sense. By defining a class of utility-perturbed Bayesian games and analyzing their equilibrium properties, the paper positions its contribution squarely within the mathematical analysis of non-cooperative games. The focus on equilibrium existence, limiting consistency, and structural stability distinguishes the approach from descriptive or application-driven behavioral models and provides a unified analytical foundation for studying utility deformation in games under incomplete information.

3. Methodology

The analysis developed in this paper is purely theoretical and model-driven. The objective is to study the equilibrium properties of a class of non-cooperative games under asymmetric information in which agents' utility functions are subject to structured parametric perturbations. The focus is on equilibrium existence, limiting consistency, and parametric stability, rather than on prediction, estimation, or empirical validation.

Strategic interaction is modeled within the standard framework of Bayesian non-cooperative games in the sense of Harsanyi. The informational structure of the game—including type spaces, priors, strategy sets, and Bayesian updating—is fixed throughout the analysis. Behavioral and perceptual effects are introduced exclusively through parametric perturbations of agents' utility functions [15]. The perturbation parameters $\lambda_i, \theta_i, \delta_i \geq 0$ are treated as normalized intensity parameters governing the magnitude of utility deformation and are not interpreted as empirically estimable quantities [16].

From a mathematical perspective, the central methodological objective is to characterize equilibrium behavior as a function of both informational primitives and perturbation parameters. Classical Nash and Bayesian Nash equilibria are recovered as limiting cases when the perturbation parameters vanish, providing a consistency benchmark for the proposed framework. Comparative-static results are obtained through the analysis of equilibrium correspondences and their continuity properties, rather than through closed-form solutions or numerical optimization [17].

The equilibrium analysis relies on standard tools from non-cooperative game theory and variational analysis. Existence of equilibrium is established under conventional compactness, continuity, and quasi-concavity assumptions. Stability and regularity properties of equilibrium correspondences are derived using continuity results for correspondences and parametric sensitivity analysis. This approach allows equilibrium outcomes to be interpreted as continuous deformations of the classical equilibrium structure induced by utility perturbations.

Any numerical illustrations included in the manuscript serve solely to visualize the equilibrium mechanisms implied by the theoretical results and to verify internal coherence. They play no role in the formal analysis and do not support inference or empirical claims. All numerical and experimental material is therefore relegated to Appendices A–C or Supplementary Materials in order to preserve the mathematical focus of the main text.

4. Mathematical Framework

This section introduces a mathematically explicit class of Bayesian games generated by parametric perturbations of utility functions. The objective is not to alter the informational structure of the game, but to define a structured deformation of classical Bayesian games

through systematic modifications of preferences, while preserving analytical rigor, equilibrium existence, and limiting consistency with standard results. Behavioral effects are introduced exclusively through utility perturbations, leaving strategy spaces, type spaces, priors, and Bayesian updating unchanged [18].

Let a Bayesian strategic game be defined by the tuple

$$G = (N, \{S_i\}_{i \in N}, \{T_i\}_{i \in N}, \{\pi_i\}_{i \in N}, \mu),$$

where $N = \{1, \ldots, n\}$ is a finite set of players, S_i is a non-empty, compact, and convex strategy set of player i, T_i denotes the type space, and μ is a common prior probability distribution on the product type space $T = \prod_{i \in N} T_i$. Payoff functions $\pi_i : S \times T \to \mathbb{R}$, with $S = \prod_{i \in N} S_i$ are assumed to be continuous and payoff-relevant. Players update beliefs according to Bayes' rule whenever possible, following the standard Harsanyi formulation.

In the classical framework, each player maximizes expected payoff with respect to beliefs consistent with Bayesian updating. In contrast, the present framework assumes that payoff evaluation is subject to systematic perturbations. These perturbations are introduced through an operator acting on the space of Bayesian games, which modifies utility functions while preserving the strategic and informational primitives of the game. Formally, for each player i, the extended utility function is given by

$$U_i(s_i, s_{-i}) = \pi_i(s_i, s_{-i}) - \lambda_i L_i(s_i) + \theta_i P_i(s_i) + \delta_i H_i(s_i),$$

where the additional terms represent structured distortions in utility evaluation.

The parameters $\lambda_i, \theta_i, \delta_i \geq 0$ are treated as normalized intensity parameters governing the magnitude of the perturbation. The function $L_i : S_i \to \mathbb{R}_+$ is assumed to be continuous, non-negative, and weakly increasing in perceived losses, capturing systematic loss sensitivity [19]. The function $P_i : S_i \to \mathbb{R}$ represents perceptual distortion and modifies subjective payoff evaluation without altering the underlying objective payoff structure. The function $H_i : S_i \to \mathbb{R}$ captures social interaction effects, such as imitation or alignment, and is assumed to increase with strategic similarity across players [20]. All behavioral functions are continuous, and their role is to generate a deformation of utility rather than to introduce additional informational content.

Collectively, the perturbation parameters define a mapping T_B, indexed by

$$B = \Big(\lambda_i, \theta_i, \delta_i)_{i \in N} \in \mathbb{R}_+^{3n},$$

which maps a classical Bayesian game G into a perturbed game $G^B = T_B(G)$ by deforming payoff functions while leaving strategy sets, type spaces, priors, and belief updating unchanged. When $B = 0$, the operator reduces to the identity and the perturbed game coincides pointwise with the original Bayesian game. The family $\{G^B\}_{B \in \mathbb{R}_+^{3n}}$ therefore embeds classical Bayesian games into a parametric family connected through continuous utility deformations.

The behavioral parameters are interpreted ordinally rather than cardinally, allowing comparative-static analysis without empirical calibration. Their role is to index the direction and magnitude of utility deformation rather than to represent measurable psychological quantities. Explicit illustrative functional forms, normalization conventions, and admissibility conditions are provided in Appendix B for expositional clarity, while preserving generality of the framework.

Within this class of games, a behavioral Nash equilibrium of the perturbed game G^B is defined as a strategy profile $s^* \in S$ such that, for all players $i \in N$,

$$U_i(s_i^*, s_{-i}^*) \geq U_i(s_i, s_{-i}^*) \text{ for all } s_i \in S_i.$$

This definition generalizes the classical Nash equilibrium by allowing preferences to vary continuously with the perturbation parameter B, while preserving the standard best-response structure and equilibrium concept.

Under standard assumptions that each strategy set S_i is non-empty, compact, and convex, and that each extended utility function U_i is continuous in the strategy profile and quasi-concave in own strategies, a behavioral Nash equilibrium exists for all admissible perturbations. This result follows directly from classical fixed-point arguments and ensures that the introduction of utility perturbations does not compromise equilibrium existence.

When all perturbation parameters vanish, that is, when $\lambda_i = \theta_i = \delta_i = 0$ for all players, the extended utility functions collapse pointwise to the classical payoff functions, and the model reduces to the standard expected-utility framework of non-cooperative game theory [1]. Limiting consistency and convergence of behavioral equilibria to classical Bayesian Nash equilibria as perturbation parameters vanish are established formally in the next section.

5. Equilibrium Properties of Behaviorally Perturbed Bayesian Games

This section presents the main mathematical results of the paper. We analyze equilibrium existence, limiting consistency, and structural stability for the class of Bayesian games generated by utility perturbation operators. The results show that utility perturbations define a mathematically well-posed deformation of classical Bayesian games, preserving equilibrium logic while introducing smooth parametric dependence on preferences.

Let

$$G^B = (N, \{S_i\}_{i \in N}, \{T_i\}_{i \in N}, \{\pi_i\}_{i \in N}, \mu, B)$$

denote a utility-perturbed Bayesian game, where N is a finite set of players, each strategy set $S_i \subset \mathbb{R}^k$ is non-empty, compact, and convex, T_i denotes the type space of player i, and μ is a common prior defined on $T = \prod_{i \in N} T_i$. The perturbation parameter

$$B = \left(\lambda_i, \theta_i, \delta_i\right)_{i \in N} \in \mathbb{R}_+^{3n}$$

indexes the deformation induced by the utility perturbation operator. Each player evaluates outcomes according to the extended utility function

$$U_i(s_i, s_{-i}) = \pi_i(s_i, s_{-i}) - \lambda_i L_i(s_i) + \theta_i P_i(s_i) + \delta_i H_i(s_i),$$

where payoff functions π_i are continuous in the strategy profile, behavioral functions L_i, P_i, and H_i are continuous in own strategies, and extended utilities U_i are quasi-concave in s_i.

We begin by establishing equilibrium existence for all admissible perturbations.

Theorem 1 (Existence of Behavioral Nash Equilibrium)**.** *Under the assumptions stated above, the utility-perturbed Bayesian game G^B admits at least one behavioral Nash equilibrium for every $B \in \mathbb{R}_+^{3n}$.*

Proof. For each player i, define the best-response correspondence

$$BR_i(s_{-i}) = \arg\max_{s_i \in S_i} U_i(s_i, s_{-i}).$$

Continuity of U_i and compactness of S_i imply that BR_i is non-empty and compact-valued. Quasi-concavity of U_i in own strategies ensures convexity of the values of BR_i, while continuity implies upper hemicontinuity. By Glicksberg's fixed-point theorem, the joint best-response correspondence admits a fixed point $s^* \in S$, which constitutes a behavioral Nash equilibrium.

We next analyze the behavior of equilibria as the perturbation parameter vanishes, establishing consistency with the classical Bayesian framework. □

Theorem 2 (Limiting Consistency with Classical Bayesian Nash Equilibrium)**.** *Let $s^*(B)$ denote a behavioral Nash equilibrium of G^B. If $B \to 0$, then any accumulation point of $s^*(B)$ is a Nash equilibrium of the underlying classical Bayesian game.*

Proof. As $B \to 0$, the extended utility functions U_i converge uniformly to the classical payoff functions π_i on the compact strategy space. By Berge's Maximum Theorem, the associated best-response correspondences converge upper hemicontinuously to those of the unperturbed game. Consequently, the behavioral equilibrium correspondence converges to the classical Bayesian Nash equilibrium correspondence, and any accumulation point of behavioral equilibria coincides with a Nash equilibrium of the underlying game. The central contribution of the paper concerns the stability of equilibria under utility perturbations. □

Theorem 3 (Structural Stability of Equilibria under Utility Perturbations)**.** *Assume, in addition, that for each player i, the extended utility function U_i is twice continuously differentiable and strictly concave in s_i. Then there exists an open neighborhood $\mathcal{U} \subset \mathbb{R}^{3n}_{+}$ of the origin such that, for all $B \in \mathcal{U}$, the game G^B admits a unique behavioral Nash equilibrium $s^*(B)$. Moreover, the equilibrium mapping*

$$B \mapsto s^*(B)$$

is continuous, locally Lipschitz, and continuously differentiable on $\mathcal{U}$.

Proof. Under strict concavity, each player's best response is single-valued and continuously differentiable. Equilibrium outcomes are characterized by the system of first-order conditions

$$\nabla_{s_i} U_i(s, B) = 0, i \in N.$$

The Jacobian of this system with respect to the strategy profile s, evaluated at the unperturbed equilibrium $(s^*(0), 0)$, is non-singular. By the Implicit Function Theorem, there exists a neighborhood of the origin in which a unique equilibrium exists and depends smoothly on the perturbation parameter. Local Lipschitz continuity follows from boundedness of the inverse Jacobian.

An immediate consequence of Theorem 3 is that, locally around the unperturbed game, all sufficiently small utility perturbations generate games with equilibrium correspondences that are topologically equivalent. □

Corollary 1 (Local Perturbation Equivalence of Games)**.** *All games G^B with B in a sufficiently small neighborhood of the origin belong to the same local perturbation equivalence class, in the sense that their equilibrium correspondences are locally homeomorphic.*

Taken together, Theorems 1–3 and Corollary 1 show that utility perturbations define a structurally stable deformation of the classical equilibrium correspondence. Equilibrium existence, uniqueness, and regularity are preserved, and equilibrium behavior varies smoothly with the perturbation parameter. Utility deformation therefore constitutes a mathematically admissible and structurally robust extension of classical non-cooperative game theory.

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6. Numerical Illustration

This section provides numerical illustrations of the equilibrium properties established in Section 5. The sole purpose of these illustrations is to visualize the qualitative mechanisms implied by the analytical results and to assess the internal coherence of the equilibrium correspondence under parametric utility perturbations. No claim of empirical validation, statistical inference, or experimental testing is made. All reported quantities are interpreted strictly as descriptive realizations of equilibrium sensitivity and finite comparative statics within the proposed theoretical framework.

The numerical illustrations are constructed from a finite set of simulated equilibrium evaluations generated under controlled variations in the informational structure and the perturbation parameter

$$B = (\lambda_i, \theta_i, \delta_i)$$

For each parameter configuration, strategy profiles are evaluated using the extended utility functions defined in Section 4, and equilibrium-relevant outcomes are obtained by iterating best-response mappings until convergence. The procedure is entirely model-driven and does not involve estimation, hypothesis testing, or data-driven optimization.

Under incomplete information and small perturbation magnitudes, numerical realizations display equilibrium profiles concentrated around conservative regions of the strategy space. In these cases, best-response correspondences are relatively flat, and equilibrium adjustments remain limited. This behavior is consistent with the analytical results, which predict continuity and stability of equilibria under small utility perturbations.

As the magnitude of the perturbation parameter increases, equilibrium profiles shift smoothly within the strategy space, even when the underlying informational structure remains fixed. From a mathematical perspective, these shifts reflect changes in the geometry of the extended utility functions and the induced deformation of best-response correspondences. Importantly, no discontinuities or equilibrium breakdowns are observed, in line with the existence and stability results established in Section 5.

Transitions between informational regimes further illustrate equilibrium sensitivity. In the absence of utility perturbations, shifts from incomplete to complete information are associated with outward movements of equilibrium strategies within the strategy space, reflecting steeper best-response correspondences and increased strategic responsiveness when informational constraints are relaxed. Under incomplete information, by contrast, equilibrium profiles remain concentrated in more conservative regions, consistent with flatter best-response mappings.

When utility perturbations are introduced, equilibrium sensitivity increases across all informational regimes, but the magnitude of the effect differs systematically. Under complete information, perturbations generate relatively modest equilibrium displacements. Under incomplete information, the same perturbation magnitudes induce substantially larger shifts in equilibrium outcomes. From a formal standpoint, this asymmetry reflects the interaction between informational opacity and utility deformation, whereby incomplete information amplifies the effect of perturbations on the equilibrium correspondence.

Additional numerical contrasts indicate that equilibrium sensitivity operates directly through deformation of utility functions rather than through auxiliary or intermediate channels. This observation is consistent with the analytical framework, in which perturbations enter exclusively through utility evaluation and induce continuous deformations of the equilibrium correspondence without altering informational primitives or strategic structure.

Overall, the numerical illustrations provide a visual representation of the theoretical mechanisms described by Theorems 1–3. Equilibrium existence and stability are preserved under all parameter configurations considered, classical equilibria are recovered as limiting cases when perturbation parameters vanish, and equilibrium outcomes vary continuously

with both informational conditions and perturbation magnitudes. These illustrations serve solely to aid interpretation of the analytical results and do not constitute empirical evidence.

All numerical inputs, parameter ranges, and illustrative datasets used in the simulations are reported in Appendix A for transparency and reproducibility. They play no role in the formal equilibrium analysis and are not used to support empirical, causal, or inferential claims.

7. Conclusions

This paper introduces and analyzes a class of Bayesian games generated by parametric perturbations of utility functions. The central contribution is mathematical: the formal definition of a utility perturbation framework and the characterization of the equilibrium properties induced by such perturbations within games under asymmetric information.

The main results show that utility perturbations preserve the core equilibrium structure of classical non-cooperative game theory. Under standard regularity conditions, behavioral Nash equilibria exist for all admissible perturbations, and the associated equilibrium correspondence converges to the classical Bayesian Nash equilibrium correspondence as perturbation parameters vanish. Utility perturbations therefore define a structurally stable deformation of the classical equilibrium framework rather than a qualitative change in equilibrium logic.

A key feature of the proposed framework is that it extends Bayesian games without altering informational primitives. Beliefs, signals, and Bayesian updating remain unchanged, while preferences are allowed to vary within a mathematically well-defined parametric family. As a consequence, deviations from classical equilibrium outcomes arise as equilibrium-consistent responses to utility deformation rather than as violations of rationality or equilibrium conditions.

From a theoretical standpoint, the analysis establishes stability, continuity, and regularity properties of equilibrium mappings under utility perturbations. The equilibrium correspondence is shown to depend smoothly on the perturbation parameter in a neighborhood of the unperturbed game, providing a rigorous foundation for comparative-static analysis and sensitivity results in non-cooperative games.

Numerical illustrations included in the manuscript serve exclusively to visualize the qualitative mechanisms implied by the analytical results and play no role in the formal derivations. Taken together, the results identify utility perturbation as a mathematically admissible and structurally robust extension of classical Bayesian games, opening avenues for further theoretical work on equilibrium deformation, stability, and equivalence in non-cooperative games under incomplete information.

From a broader mathematical perspective, the proposed framework can be interpreted as a continuous deformation of classical Bayesian games in the space of utility functions. The operator-based formulation is compatible with standard tools from fixed-point theory, correspondence continuity, and variational analysis. This perspective opens natural avenues for further theoretical work, including dynamic and repeated games, equilibrium refinement under utility perturbations, and systematic analysis of equilibrium correspondence sensitivity.

Supplementary Materials: The following supporting information can be downloaded at: https://www.mdpi.com/article/10.3390/math14030506/s1.

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Appendix A. Toy Numerical Configuration of Parameterized Equilibria

This appendix reports a purely illustrative toy numerical configuration referenced in the main text exclusively for transparency and internal coherence. The material presented here does not constitute empirical data, does not support statistical inference, and is not intended to validate, test, or falsify any hypothesis. Its inclusion is strictly auxiliary and serves only to provide a concrete numerical representation of equilibrium deformation within the class of utility-perturbed Bayesian games analyzed in Sections 4 and 5.

The numerical configuration is intentionally constructed as a minimal and stylized example rather than as an empirical dataset. A finite set of numerical configurations ($N = 20$) is generated under exogenously fixed parameter values corresponding to variations in informational regime (complete versus incomplete information) and perturbation intensity. These configurations map directly to parameter values of the utility perturbation operator defined in Section 4 and are used solely to visualize equilibrium correspondence sensitivity.

Each configuration corresponds to a numerical realization of equilibrium-relevant magnitudes under a given parameter vector. The reported quantities are abstract numerical proxies representing locations of equilibrium profiles within the strategy space and local deformation of best-response correspondences. They should be interpreted exclusively as numerical realizations of equilibrium geometry rather than as measurements of economic behavior, psychological traits, or latent variables.

No probabilistic sampling scheme, stochastic data-generating process, estimation procedure, or inferential framework is associated with this numerical configuration. The reported values are not assumed to be independent, representative, or drawn from any underlying population. Consequently, the configuration is not suitable for econometric analysis, hypothesis testing, causal interpretation, or statistical generalization of any kind.

The complete toy numerical configuration is reported below for transparency.

Table A1. Conceptual mapping between theoretical constructs and illustrative variables.

id	Info_Regime	Perceptual_Distortion	Risk_Level	Security_Level	Adjustment_Level	Loss_Index	Calibration	Group	Binary_Outcome
1	Incomplete	1	70	80	30	3.4	Low	A	1
2	Complete	0	78	80	65	4.2	Low	B	1
3	Incomplete	0	40	50	30	4.1	Low	B	0
4	Complete	1	65	88	65	5.5	High	A	1
5	Incomplete	1	70	80	30	3.7	Low	A	1
6	Complete	0	78	80	65	3.7	Low	B	1
7	Incomplete	0	40	50	30	2.7	Low	B	0
8	Complete	1	85	88	65	4.5	Low	A	1
9	Incomplete	1	70	80	30	4.3	Average	A	1
10	Complete	0	78	80	65	3.6	Low	B	1
11	Incomplete	0	40	50	30	4.2	Average	B	0
12	Complete	1	85	88	65	4.8	Average	A	1
13	Incomplete	1	70	80	30	4.6	Average	A	1
14	Complete	0	78	80	65	3.7	Low	B	1
15	Incomplete	0	40	50	30	3.7	Low	B	0
16	Complete	1	85	88	65	4.5	Low	A	1
17	Incomplete	1	70	80	30	4.0	Average	A	1
18	Complete	0	78	80	65	4.4	Average	B	1
19	Incomplete	0	40	50	30	3.9	Low	B	0
20	Complete	1	85	88	65	4.7	Average	A	1

Appendix B. Illustrative Functional Specifications and Their Theoretical Origins

This appendix provides explicit illustrative functional specifications for the behavioral components introduced in the extended utility function. Its sole purpose is to enhance mathematical transparency by exhibiting admissible representatives of the abstract functional classes implicitly assumed in the main results. None of the analytical results in the paper depend on any particular functional choice presented here.

Recall that individual utility is defined as

$$U_i(s_i, s_{-i}) = \pi_i(s_i, s_{-i}) - \lambda_i L_i(s_i, s_{-i}) + \theta_i P_i(s_i, s_{-i}) + \delta_i H_i(s_i, s_{-i}),$$

where $\lambda_i \geq 0$, $\theta_i \geq 0$, and $\delta_i \geq 0$ denote behavioral perturbation intensities.

Appendix B.1. Loss Aversion Component

The loss-aversion term is modeled as a reference-dependent disutility function consistent with prospect-theoretic preferences. A standard admissible specification is the piecewise-linear function

$$L_i(s_i, s_{-i}) = \begin{cases} 0, & \text{if } \pi_i(s_i, s_{-i}) \geq r_i, \\ r_i - \pi_i(s_i, s_{-i}), & \text{if } \pi_i(s_i, s_{-i}) < r_i, \end{cases}$$

where $r_i \in \mathbb{R}$ denotes an individual-specific reference point.

This function is continuous, non-negative, convex in losses, and weakly increasing in deviations below the reference point. Under standard assumptions on π_i, these properties ensure that the extended utility function satisfies the regularity conditions required for equilibrium existence, including continuity and quasi-concavity in own strategies.

More general convex or continuously differentiable loss-aversion functions may be employed without affecting any of the equilibrium existence, convergence, or stability results established in Section 5, provided continuity and monotonicity are preserved.

Appendix B.2. Perceptual Distortion Component

Perceptual distortion is modeled as a payoff-deformation operator acting on the objective payoff function. A simple admissible linear specification is

$$P_i(s_i, s_{-i}) = \kappa_i \, \pi_i(s_i, s_{-i}),$$

where $\kappa_i \in \mathbb{R}$ captures the intensity of perceptual amplification or attenuation.

Mathematically, this specification can be interpreted as a first-order perturbation of the payoff function. It preserves continuity and boundedness on compact strategy sets, ensuring that the extended utility function remains well behaved.

Alternative nonlinear transformations—such as logarithmic, polynomial, or sigmoid mappings—may be adopted without altering the qualitative equilibrium properties of the model, as long as continuity and boundedness conditions are maintained.

Appendix B.3. Social Interaction Component

The social interaction term is specified as a distance-based conformity function. A standard admissible form is

$$H_i(s_i, s_{-i}) = - \parallel s_i - \bar{s}_{-i} \parallel,$$

where $\bar{s}_{-i}$ denotes the average strategy chosen by the remaining players.

This specification penalizes deviations from aggregate behavior and is continuous and concave in s_i due to the negative norm. These properties ensure compatibility with the fixed-point arguments used in the equilibrium existence proofs.

More general interaction structures—such as weighted averages, network-dependent norms, or kernel-based interaction terms—can be incorporated without affecting the analytical structure of the model, provided continuity and compactness assumptions are preserved.

Appendix B.4. General Mathematical Remarks

All illustrative functional forms presented above satisfy standard regularity conditions commonly imposed in non-cooperative game theory, including continuity, boundedness on compact strategy sets, and appropriate curvature properties. Importantly, the analytical results of the paper rely only on these abstract properties and not on the specific functional expressions shown here.

When $\lambda_i = \theta_i = \delta_i = 0$ for all players, the extended utility function reduces pointwise to the classical payoff function π_i, and the model collapses to the standard Bayesian Nash equilibrium framework.

Accordingly, this appendix should be interpreted purely as an expositional complement. It illustrates that the behavioral perturbations introduced in the paper admit mathematically admissible and economically interpretable representatives without imposing any functional restrictions on the core analytical results.

Appendix C. Numerical Remarks on Equilibrium Deformation

This appendix provides additional numerical remarks concerning the deformation of equilibrium correspondences induced by utility perturbations within the class of Bayesian games analyzed in the paper. The material reported here is purely auxiliary and is included only to complement the analytical results established in Sections 4 and 5. No figures, tables, or empirical outputs are used, and nothing in this appendix plays any role in the formal derivation of equilibrium existence, limiting consistency, or structural stability.

The numerical remarks are based on simulated strategy profiles obtained under controlled variations in the informational regime and the perturbation parameter

$$B = (\lambda_i, \theta_i, \delta_i).$$

Equilibrium-relevant outcomes are computed by iterating best-response mappings under fixed parameter configurations using the extended utility functions defined in Section 4. No estimation, hypothesis testing, or statistical inference is performed, and no causal or empirical interpretation is intended.

These numerical explorations confirm the qualitative behavior implied by the analytical results. Variations in informational structure affect the geometry of equilibrium correspondences by modifying the local steepness of best-response mappings. Under complete information, equilibrium profiles exhibit greater dispersion within the strategy space, whereas under incomplete information equilibrium outcomes concentrate in more restricted regions. These effects alter the location of equilibria without affecting existence or convergence, in full consistency with Theorems 1 and 2.

Additional numerical checks indicate that perturbation components associated with curvature-inducing terms contract equilibrium correspondences by increasing local curvature of the extended utility functions, while perturbations associated with payoff amplification induce outward deformations of equilibrium mappings. In all cases, equilibrium outcomes vary continuously with the perturbation parameter, and no discontinuities or instability are observed, in line with the structural stability result of Theorem 3.

Heterogeneity across players can be accommodated by allowing agent-specific perturbation intensities, which act as scaling factors on individual best-response correspondences. Such heterogeneity affects the relative positioning of equilibrium profiles within the strategy space but does not alter existence, uniqueness (when applicable), or convergence properties.

Overall, these numerical remarks serve only to support the interpretation of utility perturbations as continuous deformations of classical equilibrium correspondences. They do not introduce new results, do not rely on visual outputs, and do not constitute empirical evidence. Their inclusion is strictly auxiliary to the formal analysis presented in the main text.

References

1. Von Neumann, J.; Morgenstern, O. *Theory of Games and Economic Behavior*, 60th Anniversary ed.; Princeton University Press: Princeton, NJ, USA, 2004. [CrossRef]
2. Harsanyi, J.C. Games with incomplete information played by "Bayesian" players I–III. *Manag. Sci.* **1967**, *14*, 159–182. [CrossRef]
3. Milgrom, P.; Weber, R.J. A theory of auctions and competitive bidding. *Econometrica* **1982**, *50*, 1089–1122. [CrossRef]
4. Stiglitz, J.E. Information and the change in the paradigm in economics. *Am. Econ. Rev.* **2002**, *92*, 460–501. [CrossRef]
5. Kahneman, D.; Tversky, A. Prospect theory: An analysis of decision under risk. *Econometrica* **1979**, *47*, 263–291. [CrossRef]
6. Camerer, C.F. *Behavioral Game Theory: Experiments in Strategic Interaction*; Princeton University Press: Princeton, NJ, USA, 2003.
7. Camerer, C.F.; Fehr, E. When does "economic man" dominate social behavior? *Science* **2006**, *311*, 47–52. [CrossRef] [PubMed]
8. Akerlof, G.A. The market for "lemons": Quality uncertainty and the market mechanism. *Q. J. Econ.* **1970**, *84*, 488–500. [CrossRef]
9. Spence, M. Job market signaling. *Q. J. Econ.* **1973**, *87*, 355–374. [CrossRef]
10. De los Reyes-Marín, Ó. Estrategias de puja en subastas con información imperfecta y sesgos cognitivos. *Econ. J. Bus. Econ. Insights* **2025**, *2*, 9–25. [CrossRef]
11. Shiller, R.J. *Narrative Economics: How Stories Go Viral and Drive Major Economic Events*; Princeton University Press: Princeton, NJ, USA, 2020. [CrossRef]
12. Cialdini, R.B. *Influence: The Psychology of Persuasion*, Updated ed.; Harper Business: New York, NY, USA, 2022.
13. Banerjee, A.V. A simple model of herd behavior. *Q. J. Econ.* **1992**, *107*, 797–817. [CrossRef]
14. De los Reyes-Marín, Ó.; Gómez-Martínez, R.; Torres-Pruñonosa, J. Estrategias competitivas en oligopolios: El impacto de la información asimétrica y el comportamiento estratégico. *Econ. J. Bus. Econ. Insights* **2025**, *2*, 47–65. [CrossRef]
15. Gennaioli, N.; Shleifer, A.; Vishny, R. *A Crisis of Beliefs: Investor Psychology and Financial Fragility*; Princeton University Press: Princeton, NJ, USA, 2018. [CrossRef]
16. Thaler, R.H. Toward a positive theory of consumer choice. *J. Econ. Behav. Organ.* **1980**, *1*, 39–60. [CrossRef]
17. Thaler, R.H.; Johnson, E.J. Gambling with the house money and trying to break even: The effects of prior outcomes on risky choice. *Manag. Sci.* **1990**, *36*, 643–660. [CrossRef]
18. Tversky, A.; Kahneman, D. Judgment under uncertainty: Heuristics and biases. *Science* **1974**, *185*, 1124–1131. [CrossRef] [PubMed]
19. Tversky, A.; Kahneman, D. Loss aversion in riskless choice: A reference-dependent model. *Q. J. Econ.* **1991**, *106*, 1039–1061. [CrossRef]
20. Young, H.P. *Individual Strategy and Social Structure: An Evolutionary Theory of Institutions*; Princeton University Press: Princeton, NJ, USA, 1998.

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