International University of La Rioja

Faculty of Education

Primary Teacher Degree

The Singapore method for teaching and learning maths: a proposal for the 2nd year students.

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Type of work:	Didactic unit
Area:	Mathematics
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Date:	31/01/2022

Abstract

This final degree work presents a didactic unit focus on the learning of the mathematics based in the Singapore method for the students of 2nd year. It is common knowledge that some students present problems while they are learning this subject with a traditional method where they just use a formula without understanding what they are really doing. This is why this method powered by Yeap Ban Har and based in different methodologies proposed by authors like Piaget, Bruner, Dienes and Skemp. The main objective of this method is to provide the students a new way to achieve new knowledges while they reinforce what they already have learnt.

An analysis of the method will be done to understand the methodology, implications and the problem solving in order to plan a didactic unit based in the learning of the multiplications for the students of 2nd year.

The purpose of this didactic unit is to improve the learning outcomes of the mathematics while developing the proposed methodology.

Key words: Singapore method, problem solving, mathematics, methodology, 2nd year.

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1. Introduction.

Mathematics are something essential in our daily life. Even if we do not realize it. As it is said, following Pastor (2008), "All the activities of our daily life need, even if they are hided, mathematics". However, it is one of the most feared subjects at school, not only for the students but also for the parents, who are worried about how their children will be able to overcome the difficulties. In order to achieve this goal, we need to know where the knowledge deficit starts and where we have to put the focus on. In some cases, the traditional teaching style cannot deal with the main problem and a new way of teaching, in which the students can understand and visualize what they are learning is require. Here is where the Singapore method takes place.

What defines this method is that the curricular design is in spiral, which means that the previous knowledges will be reinforced with the new ones, moreover the students are also given the ability of watch, handle, think, analyse and search solutions to the given problems. This method provides the teachers with new materials and tools to continue with the teaching process while they develop new strategies and opportunities for their students to learn more and better. This method is based in Jerome Bruner's theories, that said that "The teacher must provide problem situations that stimulate the children to discover the concepts, relations and procedures by themselves, as part of a whole organized" (Alonso, López y De la Cruz, 2013).

The main reason why in this work the Singapore method will be studied is to help the teachers and see how new methods can be used instead of the traditional ones or even combine with these. Apart from study this methodology, a didactic unit where we can see the advantages and disadvantages that it can bring will be done.

The idea of developing the Singapore method came out last year while a was doing the practicum. In the school where I did the internship, this methodology was using with the students of second of primary to teach the subject. What I saw was that the children understood better the explanations and was easier for them to see what

they were doing. Nevertheless, due to the covid situation the group and cooperative activities could not be done, so I was eager to investigate and experience all the chances it has.

In my case, this didactic unit will be committed to the students of second of primary, who already know how to work with this method, taking into account that they have studied it in the previous year. For the moment, we will assume that none of the students need any significant need.

The structure of the work will be the following one:

-Firstly, an analysis of the method will be done, with the main characteristics and objectives.

-Secondly, after having an idea of what the method is, a didactic unit will be developed for the students of second of primary.

-Finally, a conclusion of the research and objectives will take place.

2. Objectives of the work.

The main objective of the work will be to create a didactic unit where the Singapore method will be the teaching strategy used for the second academic year in Primary. For that purpose, a previous survey of this method will be performed with the needed implications for the didactic unit. The idea is therefore to design a realistic didactic unit that could be taught in a classroom.

The specific objectives of this work to be achieved are:

-Identify the characteristics, methodological bases and the teaching-learning model of the Singapore method and the problem solving with this group of students by revising the more relevant existing bibliography.

-Revise the existing bibliography to identify and visualize the learning difficulties that our students have when they are learning this subject.

-Design activities, resources, materials, strategies and an evaluation for the teaching-learning process.

-Create an evaluation system that allows taking into consideration all the objectives to ensure that all of them have been achieved.

3. Theoretical Framework.

In this part of the work, we will thoroughly analyse the difficulties that can be found in the mathematics' learning process, as well as the Singapore method with its methodology, advantages and disadvantages, implications and the conclusions, and finally the problem solving.

3.1. Difficulties that can be found in the mathematics' learning process.

Mathematics have been always present from the beginning of time and they have been necessary to develop different process and activities done during our life. As Santaló (1990) said, a man, without been a creator, needs the mathematical knowledge for his acts in any field of his life, and definitely, to be able to understand the world around him.

When it comes to the need of teaching maths, it is important to determine what to teach, when and how. So, it is easy for the student to understand and to apply in his life. Here is where the law and the curriculum have an important role. As it is developing in the Real Decreto 126/2014 of 28 of February, maths is one of the main subjects and its learning is one of the main objectives of the curriculum. "Develop the basic mathematical competences and starting with the problem solving which requires the realization of the elemental arithmetic operations, geometrical knowledges and estimations, as well as be able to apply them in the daily situations"(p.7).

Even if we are talking about one of the main subjects that our students must learn, the children sometimes can find different mistakes and obstacles that make this learning process somehow tedious and complicated. Those mistakes can be related to the academic failure, seen as something negative that we cannot make the most of it. From a pedagogical point of view, the mistakes can be of knowledge, of use of this knowledge, of knowing what to do and logical mistakes. Teachers have to think about a way of taking into account those mistakes. Moreover, some mistakes could be avoiding if the teacher makes an adequate progression of a concept.

In Brousseau light, "The mistake is not just the effect of the ignorance, the uncertainty, the chance, as it was believing in the empirical or behavioural theories of learning; but of the effect of a previous knowledge, which has its interest, success, and now it appears as false or just misfit. These mistakes are not accidental or unforeseen acts, its origin is set up as an obstacle " (Brousseau, 1998, p.120).

However, more and more experts believe that the problem is not in mathematics or how they are taught, but in the attitude that the students have to cope with them. That's why, the apprenticeship of a student in an emotional and affective ambit are based in the importance our own thinking and believing when we are are teaching maths. For Blanco y Guerrero (2002), the continuous failure of the students, makes themselves doubt about their intellectual capacity and they consider that their efforts are useless and feeling frustrated, so they leave when they have any difficulty. So it is necessary that teachers and pedagogues collaborate and cooperate in the affective area using projects and prevention programmes in the mathematics difficulties, in order to make this subject more attractive, so the attitudes, ideas and emotional relations that students have about it and its learning can change.

3.2. The Singapore method.

This method surged in Singapore in 1980 with the aim of dealing with some deficiencies in the mathematical area, most concrete, in the problem solving. It was an innovative approach to the elemental mathematics teaching, a study plan focused in the problem solving with figures and diagrams (Keierleber,2015). This method gave Singapore the PISA recognition, due to the students of this country were the best in the mathematics area since it was implemented. The bases of the Singapore method are in the studies of Jerome Bruner, Dines and Skempt. Who were influenced by Vygotsky's theories about development and learning. The Singapore method favours de development of processes, abilities and attitudes which foster the mathematical thinking; it is representing by making the problem solving as the focus of the process (Juarez and Aguilar, 2018).

In concrete, this proposal is based in Ashlock studies (1983), who made emphasis in how a good teaching plan must connect the learning objectives with the activities that the students do. In general, the activities are structured with the aim of structure the idea, understand it and consolidate it. Later, these keys ideas will be developed. In a traditional maths lesson, the teacher raises a problem, then the studies copy what the teacher has said and they try to solve it by doing different calculations. If their answer is the same as those of the teacher, they have done it well. "The teacher as the transceiver of knowledges and the teacher as source of answers". However, when the teacher applies the Singapore method, things change. First, the teacher raises a problem, the students assimilate what the teacher has said and they try to think how to solve it by adding their ideas. The objective is arriving to the solution by different ways with the help of the teacher without doing any calculation. In this method it is essential to think before calculating.

3.2.1 Methodological bases which support the Singapore method.

In this section different theories and methodologies which support the method will be developed. All these methodologies working together and structured over the base of solving problems are the axis of the mathematical learning. As it is highlighted by Cordano (2012), the main nucleus of this method, it is the pedagogy and the didactic materials that are used in the teaching-learning process of the mathematics. This method used in the learning of this subject has taken into account authors like: Jean Piaget, Jerome Bruner, Richard Skemp and Zoltan Dienes.

Jean Piaget.
As it is highlighted by Antonegui (2004), for Piaget the logical-mathematical knowledge is the one in which the child can connect those experiences obtained from handling objects.
Piaget stablished that there are four different stages of the child development, sensorimotor, pre-operational, concrete operation and formal operation. This explains why teachers in during the primary stage are in charge of giving the students

those opportunities and strategies to develop using motor actions and concrete material. With the aim that when the abstract stage comes they can reach the reasoning easily.

Jerome Bruner C-P-A.

The method is based on Bruner's theories, in order to get a good learning, it must be acquired a complete conceptual knowledge; the students must live these three processes: concrete, pictorial and abstract (Hilaquita, 2018).

In the first one, the concrete, the children use that material that they can touch and work with it easily, like blocks, cubes or dices.

In the second one, the pictorial, the pupils are encouraged to create an image that relate calculations or mathematical procedures to solve the problem. Here the children use bars.

In the last one, the abstract, the student is able to relate those processes with the algorithms and formulas more abstract in order to find the mathematical calculation we need.

Spiral curriculum Jerome Bruner.

As it is mentioned by Delgado, Mayta and Alfaro (2018), this methodology takes as its own support the theory of Jerome Bruner and it is focused in the CPA approach. The aim of this specific design consists of reinforce those previous knowledges that the student already has in order to achieve the new ones in a more complex, comprehensive and significate way. As Bruner said, the contents must be taught in a sequential order taking into account the development of the child. Nevertheless, if we want to continue learning more, we need to ensure those contents that the students have already studied.

 One of the applications of the Spiral curriculum could be the algebraic thought. It takes into account the previous knowledges so the students can make numerical and pictorial relations to solve the problems.

Bar model.

It is the main characteristic of the Singapore model. It is a strategic way of solving problem that will be developed in the next point. In general, we can say that it helps

to have a better understood of the concepts, visualize the problem, develops creativity and helps when planning the next step in a problem. As we will see, it has different models that allows the problem solving taking into consideration the different characteristics of the problems. For Ho and Lowrie (2014), drawing diagrams representing the relations between different quantities is helpful to understand and solve a problem that do not need to be graphics.

Zoltan Dienes. Systematic variation.

Calderon (2014) shows that another theoretical base is the Systemic variation of Dienes. This author who take as a support of his work the theories of Bruner and Piaget, he stablished two different variations about the teaching through the different ages of the students. "In our time, it is necessary to raise our children in the mathematical comprehension and its applications. This becomes an essential part of our culture " (Dienes, 1969). This means that we have to teach the students different ways to solve a problem and try to avoid the memorisation of formulas. Give the students all they need to solve the problems on their own. But first, the student needs to create the concepts and them using the adequate material, they will have to arrive to the solution. Each student will solve the problem in a different way because they perceive the characteristics of a quantity of a problem in different manners. He was the creator of the blocks and bases.

Richard Skemp.

This method is support by the theory of Skemp (1976) where he analysed the difference between the instrumental comprehension and the relational comprehension. The students have to learn this subject actively, by doing different exercises which give them experience but also reinforce the previous knowledges. They cannot learn by memorising. In the instrumental comprehension, he stablished that the students learnt step by step, using general rules a pencil and paper. In other words, they memorised the formula and then applied it in the different problems. Instead, in the relational comprehension the students have the capacity of knowing what they are doing and why they are doing it. This gives them the ability to solve any problem by using the strategies they prefer and making connection with those

things that they already know. According to Martínez (2003), shows that the knowledge of the mathematics is part of the substantial knowledge of the mathematics, in other words, the student must understand the different topics and do the procedures that help him or her to get to the right answer.

3.2.2 Teaching-learning model of the Singapore method.

A teaching learning model means the decision that the teacher makes when it comes to teach and explain the students. This method is not focused on the memorisation or in the learning or procedures o formula application, (Delgado, Mayta and Alfaro, 2018), it is focused on the strategies which help to teach in a different way. In the Singapore method, a concrete strategy could be promoting the development of procedures, attitudes and abilities that encourage the mathematical thinking, as (Juarez and Aguilar,2018) point it is known for make the problem solving the focus of the process.

Furthermore, the activities are structured around four stages, which are comprehension, consolidations, transfer and evaluation. In this way, Edge (2014) found a scheme which he updated and nowadays is helpful to suit the necessary process to teach a lesson using the Singapore method through these three activities. In the first one, comprehension. Is the first approximation of the student to a concept, using concrete material. The purpose is to arrive to a new knowledge taking into account those concepts that the student already has. It is also essential to make relations between thee new and the old concepts and relate them with those things of their routine.

In the consolidation stage, the teacher helps the student to think and reasoning while ensuring the concepts they have just learnt. In this stage the activities and games which reinforce the new knowledges are the most important.

The transfer stage occurs when the child has already assimilated the concept and now the student can apply those learnt things. Here is where the problem solving takes

place. The activities that take place here are those that extend and reinforce the learning but also those that can happen in their everyday life.

The evaluation stage is the one that measure if those learnings have been achieved by our students. It happens during the whole process of the learning since the beginning until the end. It englobes the activities, the participation of the student, the use of the materials. The main purpose is to ensure the consolidation of the learnings.

3.2.3 Components of the Singapore method.

As it has been highlighted the Singapore method is focused on the problem solving and the correct thinking of the students. For this to happen, the followed strategy must provide the development of the abilities, procedures and attitudes which developed the mathematical thinking. For Fernandez (2015), the method was developed over five components which are related between them and which all of them take place in the problem solving. These components are the concepts, the abilities, the procedures, the metacognition and the attitudes.

Concepts.
Through activities and experiences the students assimilate and relate the contents. In this group we can see different elements which are related between them. These are numerical concepts, algebraic concepts, geometric concepts, statistic concepts, probability concepts and analytic problems.

Abilities.
They are the ones that the students must achieved in order to learn and solve the problems. They can be developed when the child understands the learnings. It is more important that the child understand than learning formulas to solve the problems. In this group we can find the numerical calculation, the algebraic handling, the spatial sight and the data analysis. If the student is able to recognize the

mathematical rules, the student can also apply them and solve different problems with similar conditions to the concept learnt (Mamaní, 2017).

Procedures.

This element is essential for the comprehension of the concepts. They can be of reasoning, this means know how to analyse and create logical element; procedures of communication and cohesion, students need to know which language they have to use to explain and understand the key ideas; procedures of application and modelling, understand the problem and imagine a real situation that can happen at any moment of their lives; the thinking ability like compare and classify; the Heuristic ability, know how to visualize the solution of a problem which can not be solve directly. For Delgado et al. (2013), we have to refer to the sequence of investigation, a way to get information which is relevant for solving the problem, as well as a result of the learning.

Metacognition.

This means knows ones thinking and capacity of control the procedures and the strategies. Like thinking out loud and reflect, analyse and make changes whenever they need it. For Alonso et al. (2013), it is the capacity of the student to interrelate the acquired knowledges and to apply the learnings. The relationships with the other students and the critical thinking have an important role.

Attitudes.

This englobes the affective learning, the child's experience and thinking of the use of maths in their routine. Through conviction, interest, motivation, confidence and persistence the teacher can change the student's minds about maths and also by using those things that can be more interested, fun and significate for the children. The student not only has t be comfortable with the application of the process, but also that he or she wants to repeat it, improve it and progress (Estrada, Pizarro y Salcedo, 2019).

3.3. Problem solving.

> The problem solving is a complex activity which demands the student to have different level and types of reasoning, tries to develop competencies which allow to
> recognise and use conceptual structures and procedures of analysis of the mathematical thinking with increasing levels of complex. (UNESCO, 2001, quoted
> by Juarez M, 2018, p.79).

The aim of resolving problems using the Singapore method is to do it applying the logic instead of using a formula. This will make the students think more. In order to achieve this purpose, the method changes the type of problems that the students will solve. So, the children can apply what they do outside the classroom. Perez H. and Zambrano M. (2013) showed that the study and learning of the mathematics gives successfully to the being a science which is difficult to perform and understand in terms of education. This allows to the one that is learning to develop abilities to think encouraging in oneself the creativity to use one's strategies in the problem solving.

There are different types of activities that the children can do in class to learn maths, like basic calculations, or more complex problems. In the first one, the calculations, this type of exercise are something more methodical, but in the problem activities the students require cognitive procedures which require more complex characteristics like complex thinking, using heuristic strategies and know how to analyse quick exercises, explore concepts, procedures and mathematical relations and know how to take into account all they know about a problem to try to get to the solution.

3.3.1 Types of problems.

The Singapore method has stablished different types of problems. This classification gives the teacher the guides when the students are learning a concrete concept. So the teacher can know which one fits better with what they are studying at the moment. Yeap Ban Har, considered the world leader of this model highlighted that

"the children should have the opportunity to explore ideas when they are learning for the first time a new idea, using in class objects and concrete materials, work in team with other children. Mathematical teachers should use the children's ideas instead of just telling them what they have to memorise. Let's look for this approach". Tasks can be divided in two, routine calculations and problems. Inside the problems group, the one in which we are focusing we can find two groups, problems with a closed structure and problems with an opened structure.

Problems with closed structured.

The problems are well structured and the students can get to the solution with defined tasks using the data of the problems. In this group we can find calculation challenges, which means using a concrete operation to arrive to the solution and non routine problems, in these the students apply the heuristic characteristics. The student gets the information and understand better the way to get the more effective results in the problem (Esparza y Lobos, 2016).

Problems with an opened structure.

In this kind of problems something is missing or the procedure that the children have to follow to solve it, it is not so clear. These problems can be applied in the real life and make the students think out of the box. They can also be problems of maths investigation where the goal is to develop an extension of their knowledge, this activity helps the students to create their own strategies. Short answer problems also are part of this group, these problems can be solved very quickly but there is more than one correct answer possible. The student must visualize, think and reasoning before addressing any calculation which could get him or her to the problem solving (Ban Har, 2012).

3.3.2 Strategies and procedure to solve problems.

It is important to know that this method encourages the students to solve the problems in different ways, using techniques that normally they don't apply. This gives them more tools to solve any problem in the future. For Rodríguez (2011), it is

necessary a relation between the previous concepts and the new situations that takes place through the stimuli orientated to the playful character, with what a special motivation for the develop of the task focused on the result of the problem is created. Some strategies than can be applied are:

- Acting against the problem.
- Using diagrams.
- Create a list to solve the problems.
- Search and look for patters,
- Use trial and error.
- Solve the problem in different stages.
- Write maths reflexions.
- Create their own suppositions.

Procedure to solve a problem:

1. Analyse the problem: describe the problem with your own words and decide which is the needed information.
2. Planning the strategy: think what you can do in order to solve the problem.
3. Solve it with my strategy: solve the problem using the strategy of the previous step.
4. Answer and verify: look if the answer is correct or at least reasonable.

3.3.3 Model bars.

This is a strategy which gives the children the opportunity to picture the data and the relations between them in order to find the solution and the needed calculations to solve the problem. Kho (1987) stablished four reasons which explained why the bars should be used in the Singapore method:

1. It helps the students to have a better comprehension of the concepts.
2. Helps the students to stablish a planning for the steps that are necessary to solve the problems.
3. It is comparable and less abstract than the algebraic methods.

4. Stimulates the students to get involved in challenging problem solving.

The steps that the students have to follow are:
1. Reed the problem and reformulate the question letting a space for the answer.
2. Identify who and what.
3. Draw bars and define the parts.
4. Operate and answer with a complete sentence.

The most different thing in this method is the step where the children stablish the bars. This is helpful to represent the data and what they have to guess while they understand the problem so they can do it in the most adequate way. The visualization of the problem is the key to solve it. There are three different types of ways to carry out this model: All-Part model, Comparison model and Before and After model.

All-part model.

In this model exists a bar which represents the totality and more bars which represent the different parts. The student has to put the parts bars consecutive and above the total bar, so the child can visualize it easily. When it comes to solve the problem, the pupil can see what it is missing and what operations are required.

Comparison model.

This model shows the relation between the quantities, here instead of putting them one next to the other, a bar is above the other one, so it is easily to see the difference between them.

Before-After model.

This one is the most effective model when the problem has a previous and a post situation. The colocation of the bars is the same as the previous one, the bars are collocated one above the other one. The main difference is that the students will collocate the bars for the previous situation and for the after situation so they can see what has happened. This model is used for more complex structures.

Another study (Morin and cols, 2017) who wanted to prove the effect that were produced by the learning of the representation of bars strategy. In this study, the children have to followed 7 steps:

1. Read the whole problem.

2. Rewrite the question giving space to the answer.

3. See who was the main character of the problem.

4. Draw the bars.

5. Divided the problem to see what they have to guess.

6. Adjust the bars and calculate.

7. Write the sentence with the answer of the second step.

At the end the research showed that the students started to use cognitive strategies that they did not use before. For Forsten (2010), the youngest students can understand better the mathematical knowledges if they are taught how to use the model bars. Once they now how to use it, they can add concepts gradually in subsequent courses.

4. Contextualization.

A didactic unit about the new teaching methodology of mathematics based on the Singapore method will be carried out. It will be focused on the students of second year. It will be based in multiplications, in concrete multiplying by two, five and ten. As well as reviewing what they already know about this operation and the interiorization of the strategies which are essential to learn mathematics. The aim is that the students dominate and understand these basic multiplications with this methodology.

4.1. Features of the surroundings.

This didactic unit will be carried out in a public school of Logroño, located in the neighbourhood La Guindalera, which gives the name to the school. It is a new

neighbourhood which has been created due to the expansion of the city. It is characterised by been a residential zone.

The school started in 2009 with two classes of students of kindergarten, two teachers and the headmaster. Since then, the number of students and teachers has been increasing.

The profile type of the students is characterised by coming from families with a medium or high sociocultural level, most of the families live in the neighbourhood or in others which are near it.

4.2. Description of the center

The school is in a neighbourhood very well connected with the city centre and also surrounded by different infrastructures which give the families all the facilities they need. It is a public school which ensures the quality of the education based on the law. The students are divided in two different lines per year, in each class there are 25 students.

The school has one building divided in two, a part for the kindergarten and another part for the primary stage. In this area, there are two floors where the classes are distributed. It also has a class for the students with special needs, a library, the teacher's room, an informatics class, a class of psychomotricity, two playgrounds and a sport centre.

The centre uses an active methodology which started by a significate learning and promotes the personal autonomy of the students and the complete development of the child. The school gives to each child a personal education taking into account the rhythms of learning and the capacities of each pupil.

The school staff is characterised by been dedicated to their job and their students and been qualified. The school's methodology is orientated to be active, innovative and participative.

4.3. Characteristics of the students

The didactic unit is addressed to one of the classes of second year. The class is made up by 25 students, 13 boys and 12 girls. All of them are in the same academic level and it is a group which participates, collaborates and want to learn. In the group there is not any students who needs special needs. Nevertheless, during the teaching learning process of this didactic unit, if any need is required it will be attended. It does not matter if it is a physical or an emotional need.

5. Proposal of didactic unit.

5.1. Introduction.

This didactic unit for the students of second year is focused in the learning of the multiplications through the Singapore method. The students must Know how to do the basic operations of adding so then they can expand their previous knowledges. During this course, in terms of multiplications, the student must be able to multiply by 2, 3, 4, 5 and 10. For this reason, activities related to these contents based on the Singapore method will be developed. In addition, manipulative elements will be used in order to help the students learning. The objective is to find an innovative, collaborative and cooperative learning. In concrete, the didactic unit will be focused on learning what is the multiplication and it what it consists and learn how to multiply by 2, 5 and 10.

5.2. Justification.

The realization of this didactic unit focused in the mathematics and in particular in the Singapore method due to it is considered that the learning of the mathematics in the first courses of Primary must be very significate and relevant not only for their future life but also for the learning of other subjects. Which is why this didactic unit is proposed in order to foster this methodology. In the course of this didactic unit, the aim is that the student acquires the knowledges of the multiplications and specifically the multiplications by 2, 5 and 10.

Through the correct learning of this specific competence, the child will be able to develop other competences and will be able to use them in the current life. Teaching mathematics through this method will help to not seen them as one of the most difficult subjects at school.

5.3. Legal references.

Mathematics are part of the group of subjects called core subjects of Primary Education. The objectives, contents, the learning standards and the assessment criteria are stablished in the basic curriculum of Primary Education by the Ministry of Education, Culture and Sports, in the Real Decreto 126/2014 of the 28th of February. The Organic Law 8/2013, of the 9th of December, for the improvement of the Education Quality, the article 6 of the Organic Law 2/2006, of the 3rd of May, of the Education which has been modified in order to regulate the education process of the learning.

In the Decree 24/2014, of 13th of June, by which the curriculum of Primary Education in the autonomous community of La Rioja is stablished. In this Decree the contents of the subject and the distribution for the six courses of Primary, taking into account the level, the capacities and the age of the students. As it is said in the Decree, the acquisition of the mathematical knowledges must be done actively, this means that the students learn through the experience in situations which are familiar for them.

5.4. Objectives and competencies of the project.

5.4.1 Competencies.

In the Real Decree 126/2014 of 28th February and in the Decree 24/2014, of 13th of June, competence is conceptualized as the combination of practical abilities, knowledges, motivation, ethical values, attitudes, emotions and other social and behavioural components which work together to get an effective action. More concretely is defined as the capacities to apply in an integral way the contents of each subject and educational stage in order to realize the adequate activities and effective complex problem solving. As it is highlighted in the Decree, the competencies will be acquired through all the areas of knowledge where the concept is learnt at the same time as the procedure. The development of the competencies is helped by the design of a didactic unit. The competences in which will be the didactic unit and the activities based on are the following:

1. Linguistic communication, add the mathematical language to the student's expressions and reasoning.

2. Mathematical competence and basic competences of science and technology, developed from the specific contents of the didactic unit focused in acquired this competence.

3. Learning to learn, be able to use the mathematics in different situations and autonomously through their own experience.

4. Social and civic competences, know how to work in group and communicate with the rest of the class and respect other opinions. As well as taking into account values that can be learnt to live in society.

5. Sense of initiative and entrepreneurial spirit, use their own imagination and ideas to be able to solve the problems and to arrive to a solution on their own.

5.4.2 Objectives.

The Real Decreto 126/2024 defines the objectives as the goals that the student must achieve at the end of the educational process, as a result of the planned experiences of the teaching-learning process.

The general objective of this didactic unit consists of building the bases of a mathematical knowledge that will help them to get a complete and successful development.

The specific objectives that take place in the didactic unit are based in the Decree 24/2014, of 13th of June:

1. Learn the multiplications like if they were a repetitive sum using the proposed methodology.

2. Know how to multiply without taking into account the order of the numbers.

3. Memorising the multiplication tables.

4. Learn to make their own multiplication table.

5. Make operations using the didactic material.

6. Develop the practical learning of the concept through the mathematical thinking.

7. Work together in order to consolidate the new learnings and to make them more significate.

8. Identify the vocabulary related to the didactic unit.

9. Make their own strategies to solve the problems.

10. Acquire the basic competencies of the subject with the best possible results.

11. Create a comfortable environment for learning and motivate the students.

5.5. Contents.

In this section the contents of the didactic unit will be presented. They are in relation with the Real Decreto126/2014, of 28th of February, in which the basic curriculum of the Primary Education in stablished and also the Decree 24/2014 of the 13th of June, by which the curriculum of the Primary Education in the autonomous community of La Rioja.

-Programme the problem-solving process and use their strategies.

-Analysis and comprehension of the statement.

- Initiation to the multiplication as a sum of summands. Multiplication tables.

-Elaboration of their own strategies.

-Oral explanation of the process done.

-Construction and memorisation of multiplication tables.

5.6. Methodology.

Regarding to the methodology that will be carried out in this didactic unit, the purpose is that the student himself get involved in his own learning, which means that he learns by his own experience. Each student is in charge of adding the new learning to the previous concepts they already have.

Furthermore, this is a participative and active methodology where the students interact with each other so they can increase their knowledges. They teacher is the one that gives the students activities and guides them in their learning process, but is the student, who is the main character of the learning, the one that will do the

activities and experience with them so he can reinforce what he is learning and remember the main ideas for future learnings.

The didactic unit is focused on the multiplications, in concrete in multiplying by 2,5 and 10. It will be divided in 5 sessions, each one last approximately one hour where the students will do different activities and learn new concepts.

In the next section, the activities that will be carried out in each session, as well as the objectives and the competencies to reach.

5.7. Sessions.

Session 1. Introduction to multiplications.

This session is an introduction for the students to the world of the multiplications. What implies to teach the students different strategies to start to multiply. The aim of this session is to teach the students how to count in different ways by creating groups. In the previous units the students have learnt the numbers from zero to one thousand, to add and to subtract. Once they have achieved these knowledges, they can learn the multiplications.

-Objectives and competencies.

The competencies related to the following objectives are: linguistic communication, mathematical competence, learning to learn and sense of initiative and entrepreneurial spirit.

The objectives are:

1. Learn the multiplications like if they were a repetitive sum using the proposed methodology.

5. Make operations using the didactic materials.

6. Develop the practical learning of the concept through the mathematical thinking.

8. Identify the vocabulary related to the didactic unit.

9. Make their own strategies to solve the problems.

10. Acquire the basic competencies of the subject with the best possible results.

11. Create a comfortable environment for learning and motivate the students.

-Materials and spaces.

To carry put this session in class, it is necessary to use the the tiles which are part of the manipulative materials of the Singapore method.

-Description of the session.

The session starts by writing in the board a succession of numbers and asking the students to complete with the numbers that are missing. Then, the teacher asks them, which is the pattern that the succession has. The purpose is to see what the children already know and encourage them to participate.

Then, the teacher makes groups with the tiles, each group has to have the same number. The children copy the groups with their own tiles. Next, the students have to think different ways to count how many there are in total. For example, the teacher shows 4 groups of 2 tiles each. The child can add the groups (2+2+2+2) or can count two by two (2,4,6,8) to find the total files.

Finally, the students do some activities related with the multiplication as a repetitive add in order to acquire the new knowledges and put in practice what they have learnt. In this sheet, the students do activities where he or she has to complete the blank spaces related with multiplications and another where has to observe and think. See annex 1 with the activity.

-Evaluation of the activity.

The teacher carries out an observational evaluation through the course of the session and then will add the information in the evaluation record to assess the didactic unit (Annex 2). The students will have to give to the teacher the sheet done for its correction and qualification.

Session 2. Multiply no matter the order is and learn the steps of the problem solving.

Once the students know how to multiply as a repetitive add, it is time that they learn that they can multiply in several orders. To support this concept, the teacher uses a visual and manipulative material to explain it easily.

-Objectives and competencies.

The competencies related to the following objectives are: linguistic communication, mathematical competence, learning to learn, social and civic competencies and sense of initiative and entrepreneurial spirit.

The objectives are:

2. Know how to multiply without taking into account the order of the numbers.

5. Make operations using the didactic material.

6. Develop the practical learning of the concept through the mathematical thinking.

7. Work together in order to consolidate the new learnings and to make them more significate.

8. Identify the vocabulary related to the didactic unit.

9 Make their own strategies to solve the problems.

10. Acquire the basic competencies of the subject with the best possible results.

11. Create a comfortable environment for learning and motivate the students.

-Materials and spaces.

To carry put this session in class, it is necessary to use a die, the cubes and the table which are part of the manipulative materials of the Singapore method.

-Description of the session.

First, the teacher shows the student two different multiplications, for example (2x3 and 3x2) and asks the students if they think that they are the same multiplication, to solve the question the teacher uses the cubes and the chart. With the visual material the teacher shows that we can joint factors in different ways and the answer will always be the same. Then the teacher makes different examples that the students have to solve. (Annex 3).

In the second part of the session, the students learn how to solve problems and the steps they have to follow. The teacher explains the four steps by using a problem as an example. The problem in each step could be:

1. Analyse the problem. Mary has 40 stamps and want to swap them for marbles. They give her one marble for 10 stamps. How many marbles will them give to her?

2. Planning their own strategy. We can make groups of 10 stamps to see how many marbles will Mary get.

3. Solve with their own strategy. Make the groups of the stamps.

4. Answer and verify. 10 stamps= 1 marble, 20 stamps= 2 marbles; 30 stamps= 3 marbles; 40 stamps= 4 marbles. We can also simplify it with a multiplication.

Then, the students will have to solve a problem on their own taking into account the previous steps. The problem they have to solve is: Alex has bought 10 rubbers, 10 pencils and 10 sharpeners. In order to share them with their mates, he has made groups that each has a rubber, a pencil and a sharpener. How many groups has he made? Draw them.

Once they have finish in pairs, they will roll a die of 6 sides and they will have to create groups as many groups of cubes as the die points. Then they have to write the operation in order to create those groups.

-Evaluation of the activity.

As in the previous session, the teacher carries out an observational assessment which will be written in the evaluation record (annex 2). The teacher observes if the student is doing the activity, helping his or her pair, or any other situation that can take place in the course of the lesson.

Session 3. Let's multiply by 2!

Now that the students already know what multiplications are, it is time to start learning the multiplication tables. Firstly, to make it easy, we start with the 2 chart. Through the learning of this chart, they children will be able to make the next charts and it also will be helpful to do different activities.

-Objectives and competencies.

The competencies related to the following objectives are: linguistic communication, mathematical competence, learning to learn, social and civic competencies and sense of initiative and entrepreneurial spirit.

The objectives are:

3. Memorising the multiplication tables.

4. Learn to make their own multiplication table.

5. Make operations using the didactic material.

6. Develop the practical learning of the concept through the mathematical thinking.

7. Work together in order to consolidate the new learnings and to make them more significate.

8. Identify the vocabulary related to the didactic unit.

9. Make their own strategies to solve the problems.

10. Acquire the basic competencies of the subject with the best possible results.

11. Create a comfortable environment for learning and motivate the students.

-Materials and spaces.

To carry put this session in class, it is necessary to use the cubes, the tiles, a chart and a coloured cardboard.

-Description of the session.

The students are divided in groups of four. Each group is given a cardboard and the students also need their cubes and their chart. Each group with the help of their tiles have to make a chart which shows the 2 multiply chart. They have to create a chart from multiply 2 times 1 to multiply 2 times 10. First, they have to put the tiles in the table (annex 4). Once the teacher verifies that it is well done, they have to write down with numbers and the sings of the multiplication, see annex 5. When everything is done, they can take out the cardboard and draw a chart, complete it with the 1 and 2 multiplication chart. The final objective is that each group creates a chart with all the multiplications (annex 6).

In the second part of the lesson, the groups are given different multiplications that they have to do following these instructions:

-A member of the group has to create groups with the cubes.

-Another member has to solve the multiplications using the tiles and the chart.

-The third member has to solve the multiplications.

-The last member has to prove that the results of the other members are right by using the multiplication chart they they have done in the first part of the class.

The multiplications that the students have to do are: 3x2; 2x8; 2x7; 9x2.

-Evaluation of the activity.

The evaluation of this session will be done through the observation and the qualification of the operations done by each group. The teacher observes and notes the behaviour and the development of the activity.

Session 4. Multiply by 5 and 10.

With this session the students continue with the creation of the chart and learning the multiplications. Developing all the multiplications the children will be able to do all the activities which implies these operations and also learning other concepts which need multiplications as a basic concept.

-Objectives and competencies.

The competencies related to the following objectives are: linguistic communication, mathematical competence, learning to learn, social and civic competencies and sense of initiative and entrepreneurial spirit.

The objectives are:

3. Memorising the multiplication tables.

4. Learn to make their own multiplication table.

5. Make operations using the didactic material.

6. Develop the practical learning of the concept through the mathematical thinking.

7. Work together in order to consolidate the new learnings and to make them more significate.

8. Identify the vocabulary related to the didactic unit.

9. Make their own strategies to solve the problems.

10. Acquire the basic competencies of the subject with the best possible results.

11. Create a comfortable environment for learning and motivate the students.

-Materials and spaces.

To carry put this session in class, it is necessary to use the cubes, the tiles, a chart and a coloured cardboard.

-Description of the session.

As in the previous session, the students are divided in the same groups. The purpose is to continue with the activity done. To continue with the previous work, the students with the help of the tiles and the chart they have to do the 5 time table and the 10 time table. Once the teacher has proved that the chart done with the tiles, they can write down the multiplications and finally complete the chart of all the multiplications, now they can write the 5 and the 10 time table.

In the second part of the lesson the students in groups have to create two problems. In both of them a multiplication is required, in one of them they have to use the 5 table and in the other one the 10 table. The teacher has to make sure that the problems are right. Then the teacher collects all the problems and switch them between the different groups. All the groups have to solve all the problems written by the others groups and also the ones that they have done. The students have to solve the problems following the steps to solve the problems that they have learnt in the previous sessions.

-Evaluation of the activity.

The evaluation will be a combination between an observational assessment which will be registered in the evaluation record and a marking of the problems done during the lesson.

Session 5. The last session.

Taking into account that this is the last session of the didactic unit, this session will be dedicated to review the knowledges learnt during the course of the didactic unit and to complete a sheet where the teacher can evaluate if the learnings and the objectives have been acquiring by the students.

-Objectives and competencies.

The competencies related to the following objectives are: linguistic communication, mathematical competence, learning to learn, social and civic competencies and sense of initiative and entrepreneurial spirit.

The objectives are:

1. Learn the multiplications like if they were a repetitive sum using the proposed methodology.

2. Know how to multiply without taking into account the order of the numbers.

3. Memorising the multiplication tables.

4. Learn to make their own multiplication table.

5. Make operations using the didactic material.

6. Develop the practical learning of the concept through the mathematical thinking.

7. Work together in order to consolidate the new learnings and to make them more significate.

8. Identify the vocabulary related to the didactic unit.

9. Make their own strategies to solve the problems.

10. Acquire the basic competencies of the subject with the best possible results.

11. Create a comfortable environment for learning and motivate the students.

-Materials and spaces.

The board of the class, the sheets given by the teacher and the cubes.

-Description of the session.

Firstly, the teacher reviews all the concepts taught during the didactic unit while answering all the doubts that the students have and resolving the problems and activities where they have troubles.

Once all the doubts are solved the teacher gives to the students a sheet with different activities and problems that they have to solve individually. To help themselves the students can use the cubes. In the sheets there will be activities of multiplying as a repetitive add, multiply in any other and multiplications by 2, 5 and 10. (Annex 7).

-Evaluation of the activity.

The evaluation of the activity will be a qualification of the sheet done by the students.

5.8. Timing and Schedule.

The didactic unit has been developed focused on the learning of the multiplications of the students of second year. This didactic unit will be developed in the first term, in concrete just after the learning and reviewing of the numbers from 0 to 1000, which includes the learning of the adding and subtracting. As it is one of the core subjects, the students will have this subject four of the five days of the week, as it is stablished in the Real Decreto 126/2014, of the 28th February, where the basic curriculum of the Primary Education is stablished. Each session has a duration of an hour. Due to the didactic unit has five sessions, the course of its learning will last a week and an extra day of the next week.

5.9. Attention to Diversity.

In the proposed didactic unit, it has not been identifying any student with a specific learning need. Nevertheless, this methodology gives the student the chance to work at their own rhythm and it also gives the teacher the opportunity to observe all the students and be aware that all of them acquire the knowledges. It is obvious that not all the students will get the competencies and objectives and the same time, some of them will do it very quickly but maybe a few of them need more time, attention and

practice to arrive at the same point. The aim of doing these different activities is that all of them give the students the time and circumstances they need to carry them out.

5.10. Assessment system.

The evaluation takes place during the whole course of the didactic unit. It is important to be always evaluation not just for marking the students but to know when to take action whenever any student need it. This means that it is a continuous evaluation. During the lessons, the teacher is watching the students and asking them questions to analyse their implication with the subject and more important to know if they have any problem.

The evaluation takes place in three different moments. At the beginning of the first session, while the teacher is doing the exercise with the students, an initial evaluation takes place. This serves the teacher to know the level in which the students start. During the course of the teaching process the continuous evaluation properly said takes place. Through the observation, the teacher can know the evolution and development of the students. All what the teacher sees during the lessons is note on the evaluation record (annex 2). The third moment is the final evaluation, this is the final activity that the students do during the didactic unit (annex 7). Here is where the learning outcomes are evaluated and where the we can see if the objectives have been acquire or not.

Table 1. Assessment system.

Assessment criteria	Learning outcomes
1. Does basic multiplications using different processes and strategies.	1.1 Initiates in the realization of multiplications. 1.2 Initiates in the construction of multiplication charts. 1.3 Elaborates multiplication charts by memorising the results. 1.4 Memorise the multiplication charts.
2. Knows, elaborates and uses basic strategies to apply them to the problem solving.	2.1 Creates and memorises multiplication charts. 2.2 Uses basic multiplications to calculate.

3. Identifies and solves problems from the normal life, creating connexions between reality and mathematics, using adequate knowledges to solve problems.	3.1 Uses the multiplications to solve the problems. 3.2 Solves problems which implies just one ordering using and creating different strategies. 3.3 Initiates in the reflexion of the problem solving.

(source: own elaboration)

5.10.1. Assessment criteria

ASSESSMENT CRITERIA	Objective	Learning outcome	Session/ Activity
Criterion 1	1,2,3,4,5,8,10,11	LC,MC,LL	All
Criterion 2	1, 2, 3,4,5,8,10,11	LC,MC,LL	All
Criterion 3	1, 2, 5,6,7,8,9,10,11	SC,LC,MC,LL	All

5.10.2. Assessment tools

-The teacher has an evaluation record for each student where notes the interventions or anything which considers important to assess them.

-Diary observation in class, participation and oral activities done in class and the respect of co-habitation rules.

-The exam or final evaluation, with different activities similar at the ones that have been done during the didactic unit.

-The evaluation of the activities carried out during the topic.

6. Conclusions

The main objective of this TFE was to design a didactic unit where the Singapore method could be applied in the course of the second year of Primary. For this purpose, a concrete design of a didactic unit for the learning of an essential part of the mathematics has been done, the multiplications. Group and individual activities

have been proposed, as well as different activities that can help to get the knowledges.

In order to achieve this goal, it is necessary to give analysed the specific objectives which were raised at the beginning of the work:

With regard to *identify the characteristics, methodological bases and the teaching-learning model of the Singapore method and the problem solving with this group of students by revising the more relevant exiting bibliography,* we can say that after the investigation done in the the theoretical framework, different authors agree that mathematics is not only a subject that has to be studied at the school, it goes beyond of knowing how to add or subtract, in contrast it is essential for the children to develop a critical thinking which will be useful in the future life. Focusing in the method, it is obvious that is a new way of teaching which motivate the students to learn by manipulating the cubes while they learn new concepts and revise the previous knowledges and more importantly, this method aims the students to think logically and to solve problems in different ways. Nevertheless, for some students this method could be tedious, if they find this methodology imaginative and difficult to see the solution, they could prefer a traditional method where they only have to use the formulas and the rules already stablished.

With regard to *design activities, resources, materials, strategies and an evaluation for the teaching-learning process.* Thanks to the use of the manipulative materials that this method has and adding new ones to create activities, the goal was to motivate and encourage the student's mathematical learning. By carrying out dynamic and participative activities focused in the teaching of the multiplications. Using the Singapore method, resources like cubes or tiles in order to reach the children's learning.

Regarding to *create an evaluation system that allows taking into consideration all the objectives to ensure that all of them have been achieved.* Knowing which was the main objective that we want the students to achieve, specific objectives that help to get the purpose were specified in order to create a didactic unit based in what the students need in the moment of learning.

Finally, point that the main objective of designing a didactic unit to work with the Singapore method in the 2nd year of Primary Education has been achieved. T o get it, primarily the main difficulties that students can have in the learning process have been analysed, also the new method has been discussed, focusing in the methodology, bases and implications in order to finally design a didactic unit adapted for the students.

7. Final considerations

After the analysis done of the Singapore method, it may be said that is a dynamic method, which starts from the concrete knowledge and ends in the abstract knowledge. It also involves the child it his or her own learning since the beginning. In a traditional methodology, sometimes the students just do the calculations by using a formula without taking into account what they are doing and why. On the contrary, this method encourages the student to think and create his or her own strategies to solve problems, and more importantly, it has to be emphasized, that this methodology allows the student to revised what they already know and apply these concepts to those concepts that they are learning.

Even if this method is very useful there are some limitations that can be found in the development of the didactic unit. Firstly, the teacher who is going to apply this method need to be training in advance in order to know how to use the materials and how to encourage and guide the students in their learning. Talking about the student's learning, for some of them it can be tedious to think different ways to solve a problem because they cannot find more than one answer and finally, if this methodology is only used during Primary Education then when the students go to high school, they can have troubles in changing they way of learning.

Nevertheless, even if there are some disadvantages, this method is very useful for all students due to it can be adapt to their own learning and their needs in the different stages of learning. If the students know how this method work since they start learning mathematics in Pre-primary Education, they won't have any problem with

the methodology, the materials and in knowing how to think to create strategies and find more than one correct answer. For this reason, it would be interested to see this methodology throughout all the stages of a class to see how it develops, how it encourages the students and evaluate it to be able how it evolves sand improves the learning of the mathematics.

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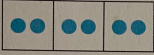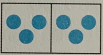
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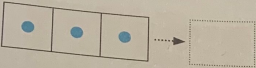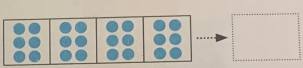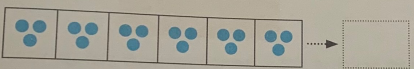Skemp, R. (1976). Relational understanding and instrumental understanding. Mathematics Teaching.

9. Annexes

9.1 Annex 1: Activities about the repetitive add.

1. Use the table and complete each numerical expression:

2.Complete the blank spaces. Use the tables to help yourself.

$1 + 1 + 1 =$ ....

$1 \times 3 =$ ....

$6 + 6 + 6 + 6 =$ ....

$6 \times 4 =$ ....

$3 + 3 + 3 + 3 + 3 + 3 =$ ....

$3 \times 6 =$ ....

3. Complete the blank spaces.

$5 + 5 + 5 + 5 =$ ....

.... $\times 4 =$ ....

$6 + 6 + 6 =$ ....

.... $\times 3 =$ ....

4. Observe the add: 3+3+2 Do you think that is it possible to express it as a multiplication? Explain your answer.

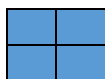## 9.2 Annex 2: evaluation record.

Name of the student:

Activity:

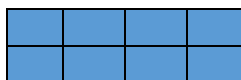|	1	2	3	4	5	6	7	8	9	10
knows how to use the tables and the manipulative materials.										
Use different strategies to solve the problems.										
Follows the proposal steps to solve the problems.										
Use the created tables and materials to solve the problems.										
Knows how to work in team.										
Solve the problem in a reasonable way.										
Participates actively in class.										
Has an empowering attitude towards the learning.										

Global valuation:

Comments and observations:

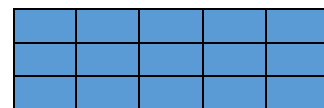## 9.3 annex 3: Activities to practise the new knowledges.

_____ x _____ = _____
number   squares   result
of lines in a line

_____ x _____ = _____
number   squares   result
of lines in a line

_____ x _____ = _____
number   squares   result
of lines in a line

_____ x _____ = _____
number   squares   result
of lines in a line
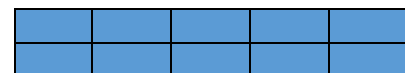
9.4 Annex 4: tiles and chart.

9.5 Annex 5: the twos time table.

2x1=2
2x2=4
2x3=6
2x4=8
2x5=10
2x6=12
2x7=14
2x8=16
2x9=18
2x10=20

9.6 Annex 6: the final multiplication chart.

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

9.7 Annex 7: final evaluation.

Activity 1: Sally has 3 boxes and each one there are six balls. How many balls are there

in total? Draw the balls creating groups and solve the operation.

Activity 2. Look to the chart, then imagine that it changes its position, from vertical to horizontal. Which are the two operations which correspond to the chart? Marc the correct   answers.
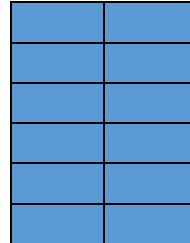
2x6=12
12-6=12

6+4=12
6x2=12

5x6=12
2x6=12

6x2=12
2x6=12

Activity 3. Tom has five boxes of tiles and each one has six tiles. Greg has two boxes of tiles and each one has ten. Who has more tiles?

_____x_____=

_____ has more tiles.

_____x_____=

Activity 4. Complete the chart.

1x10		3x10	4x10		6x10	7x10		9x10	
10		30		50	60		80		100

Activity 5. Create a problem and solve it. The operation must be 2x5.