

Temporal Information Processing and Stability Analysis of the MHSN Neuron Model in DDF

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Abstract — Implementation of a neuron like information processing structure at hardware level is a burning research problem. In this article, we analyze the modified hybrid spiking neuron model (the MHSN model) in distributed delay framework (DDF) for hardware level implementation point of view. We investigate its temporal information processing capability in term of inter-spike-interval (ISI) distribution. We also perform the stability analysis of the MHSN model, in which, we compute nullclines, steady state solution, eigenvalues corresponding the MHSN model. During phase plane analysis, we notice that the MHSN model generates limit cycle oscillations which is an important phenomenon in many biological processes. Qualitative behavior of these limit cycle does not changes due to the variation in applied input stimulus, however, delay effect the spiking activity and duration of cycle get altered.

Keywords — Distributed Delay, Dynamical System, Eigen Value, Hybrid Spiking Neuron, ISI Distribution, Neuromorphic Engineering, Phase Plane Analysis.

I. INTRODUCTION

THE human brain is the most complex dynamical system with 11th order of neuron and 15th order of total synaptic connections among themselves [1, 2, 3, 4]. Researchers are involve in understanding the biophysical dynamics and information processing functionality of the brain since last six decades, so that an artificial brain like structure can be implemented at software as well as at hardware level [5, 6, 7, 8, 9, 10, 11, 12]. Hardware level implementation of the artificial brain lies in the domain of Neuromorphic engineering in which one implements few functionality of a real neuron on a chip [9, 13]. Integrate-and-fire (IF), leaky integrate-and-fire (LIF) and hybrid spiking neuron models are suitable choice for chip level implementation due to their simple mathematical treatment [3, 9, 13, 14]. These neuron models are threshold based models [8, 15, 16]. For hardware level implementation of a threshold based neuron model, one has to focus on two prime issues, namely, (i) How to implement the reset condition of membrane potential after spiking activity? (ii) How to maintain the threshold value and control the variability of threshold value due to the rise in temperature of the chip [3, 5, 9, 17]. Izhikevich two dimensional hybrid spiking neuron model has the mechanism to overcome the first issue at its hardware level implementation [3]. But it is still a challenging problem to fix the variability in threshold value at implementation level, which generates a large variation in spiking pattern.

A varying time delay occurs in neuronal information processing system due to the varying length of axons, structure of neurons and flow of neurotransmitters form one region to another region into the nervous system [10, 18, 19]. Distributed delay framework suggested by Mar et. al. [6] has the virtue to incorporate the varying time delay in a neuron model in terms of distributed delay kernel functions. Karmeshu et. al.

[20, 21] has investigated the LIF model with stochastic input stimulus in DDF and has explained many interesting neurological phenomenon such as transient bimodality in spiking activity of a neuron.

Bhati et. al. [22] has calculated the analytical explicit expressions for membrane potential and recovery variable in hybrid spiking neuron model with constant input stimulus in DDF with minor modification. Choudhary et. al. [23] has investigated the spiking activity of the modified hybrid spiking neuron model in DDF (MHSN model) with four different kinds of input stimulus in DDF and noticed very small variability in its spiking pattern against large variation in input stimulus.

In this article, we perform the stability analysis and temporal information processing capability of hybrid spiking neuron model in DDF. The article is organized into five sections. Section II describes the MHSN model in detail. Section III investigates the temporal information processing of the MHSN model. Section IV is devoted in stability analysis of the MHSN model. In this section, we also perform the phase analysis of the model. Last section V contains the conclusion and scope for the future research work.

I. THE MODIFIED HYBRID SPIKING NEURON MODEL IN DDF

Izhikevich [3] has suggested a family of threshold based neuron models governed by two state variables, namely membrane potential (V) and recovery variable (U), in term of a system of coupled differential equations.

$$\frac{dV}{dt} = f(V) - U(E - V) + I \quad (1)$$

$$\frac{dU}{dt} = a(bV - U) \quad (2)$$

with after spiking reset condition (threshold constraint): if $V \geq V_T$ then $V \leftarrow V_R$ and $U \leftarrow U + U_I$.

Here, $f(V)$ is membrane potential-current relationship function. I , E , V_T and V_R are input stimulus, reversal potential, membrane potential threshold and resting potential, respectively. a and b are model parameters.

Following Bharti et. al. [22], the MHSN model takes the form

$$\frac{dV}{dt} = \int_0^t K(t - \tau)V(\tau)d\tau - U + I \quad (3)$$

$$\frac{dU}{dt} = a(bV - U) \tag{4}$$

Here, $K(t)$ is the distributed delay kernel function. There may be a number of choices of $K(t)$, but Bharti *et. al.* [22] has studied the MHSN with exponential distributed delay kernel function, which is also known as weak delay. In presence of exponential distributed delay kernel function MHSN model becomes

$$\frac{dV}{dt} = \int_0^t \eta e^{-\eta(t-\tau)} V(\tau) d\tau - U + I \tag{5}$$

$$\frac{dU}{dt} = a(bV - U) \tag{6}$$

with initial condition: $V = V_0$ and $U = U_0$ at $t = 0$. Here η is the delay parameter. Eqns. (5) and (6) make a system of coupled integro-differential equations. Its investigation is a complex task due to the presence of integro-differential term. By applying Laplace transform and inverse Laplace transform, Bharti *et. al.* [22] has calculated the explicit expression for V and U with constant I which given as below.

Case: [1] when $a \geq 4b$, the membrane potential and recovery variable takes the form

$$V(t) = A_1 + B_1 e^{\alpha_1 t} + C_1 e^{\beta_1 t} \tag{7}$$

where

$$A_1 = \frac{aI}{\alpha_1 \beta_1}, \quad B_1 = \frac{\alpha_1(\alpha_1 V_0 + aV_0 - U_0 + \eta + I) + aI}{\alpha_1(\alpha_1 - \beta_1)}$$

$$C_1 = -\frac{\beta_1(\beta_1 V_0 + aV_0 - U_0 + \eta + I) + aI}{\beta_1(\alpha_1 - \beta_1)}$$

and

$$U(t) = A_2 + B_2 e^{-\eta t} + C_2 e^{\alpha_2 t} + D_2 e^{\beta_2 t} \tag{8}$$

where

$$A_2 = \frac{ab(V_0 + I)}{\alpha_2 \beta_2}, \quad B_2 = \frac{abI}{(\alpha_2 + \eta)(\beta_2 + \eta)}$$

$$C_2 = \frac{\alpha_2^2 U_0 + [ab(\eta + V_0 + I) + U_0 \eta] \alpha_2 + ab\eta(V_0 + I)}{\alpha_2(\alpha_2 + \eta)(\alpha_2 - \beta_2)}$$

$$D_2 = -\frac{\beta_2^2 U_0 + [ab(\eta + V_0 + I) + \eta U_0] \alpha_3 + ab\eta(V_0 + I)}{\beta_2(\beta_2 + \eta)(\alpha_3 - \beta_3)}$$

Case: [2] when $a < 4b$, the membrane potential and recovery variable takes the form

$$V(t) = A_3 + e^{\alpha_3 t} [B_3 \cos \beta_3 t + (\alpha_3 B_3 + C_3) \sin \beta_3 t] \tag{9}$$

where

$$A_3 = \frac{aI}{\alpha_3^2 + \beta_3^2}, \quad B_3 = \frac{(\alpha_3^2 + \beta_3^2)V_0 + aI}{\alpha_3^2 + \beta_3^2}$$

$$C_3 = \frac{(\alpha_3^2 + \beta_3^2)(aV_0 - U_0 + \eta + I) + 2a\alpha_3 I}{\alpha_3^2 + \beta_3^2}$$

and

$$U(t) = A_4 + B_4 e^{-\eta t} + e^{\alpha_4 t} [C_4 \cos \beta_4 t + (\alpha_4 C_4 + D_4) \sin \beta_4 t] \tag{10}$$

where

$$A_4 = -\frac{(\eta + 2\alpha_4)[ab(\eta + V_0 + I) + \eta U_0] + U_0(\alpha_4^2 + \beta_4^2)}{\alpha_4(\alpha_4^2 + \beta_4^2)[\alpha_4^2(2 - \eta - 2\alpha_3) + 2(\beta_4^2 + \eta^2 + 2\eta\alpha_4)]}$$

$$B_4 = \frac{ab(\eta + V_0 + I) + \eta U_0}{(\alpha_4^2 + \beta_4^2)} + \frac{(\alpha_4^2 - 2\eta)[(\eta + 2\alpha_4)\{ab(\eta + V_0 + I) + \eta U_0\} + U_0(\alpha_4^2 + \beta_4^2)]}{(\alpha_4^2 + \beta_4^2)[\alpha_4^2(2 - \eta - 2\alpha_3) + 2(\beta_4^2 + \eta^2 + 2\eta\alpha_4)]}$$

$$C_4 = -\frac{ab(\eta + V_0 + I) + \eta U_0}{(\alpha_4^2 + \beta_4^2)} + \frac{(2\alpha_4 + \beta_4^2)}{\alpha_4(\alpha_4^2 + \beta_4^2)^2} \frac{[(\eta + 2\alpha_4)\{ab(\eta + V_0 + I) + \eta U_0\} + U_0(\alpha_4^2 + \beta_4^2)]}{[\alpha_4^2(2 - \eta - 2\alpha_3) + 2(\beta_4^2 + \eta^2 + 2\eta\alpha_4)]}$$

$$D_4 = ab(V_0 + I) + \frac{\beta_4^2 [(\eta + 2\alpha_4)\{ab(\eta + V_0 + I) + \eta U_0\} + U_0(\alpha_4^2 + \beta_4^2)]}{\alpha_4(\alpha_4^2 + \beta_4^2) [\alpha_4^2(2 - \eta - 2\alpha_3) + 2(\beta_4^2 + \eta^2 + 2\eta\alpha_4)]}$$

It is a too complex to further investigate the MHSN model with integro-differential equation term. In presence of integro-differential term, the membrane potential evolution process $V(t)$ becomes a non-Markovian process. In order to transform $V(t)$ into a Markovian process, the MHSN model can be extended in infinite dimensional space [20, 24]. Following Choudhary *et. al.* [23], substitution of

$\int_0^t \eta e^{-\eta(t-\tau)} V(\tau) d\tau$ by $X_0(t)$ in the MHSN model and after some simplification, the three dimensional MHSN model with exponential distributed delay kernel function in extended space takes the form:

$$\frac{dV}{dt} = X_0 - U + I \tag{11}$$

$$\frac{dU}{dt} = a(bV - U) \tag{12}$$

$$\frac{dX_0}{dt} = \eta(V - X_0) \tag{13}$$

Eqns. (11), (12) and (13) make a system of coupled linear differential equation. Its analytical as well as simulation based investigation is an easier task. We consider the above stated three dimensional MHSN model in our investigation. We perform the simulation based study to investigate the temporal information processing capability of the MHSN model in the next section.

II. TEMPORAL INFORMATION PROCESSING

Spiking activity is an essential feature in neuronal information processing [14, 25]. A neuron encodes processes and transmits information in the term of epoch of membrane potential (spike) [4, 14, 25, 26]. Rate coding and temporal coding are two important encoding strategies in neuronal information processing [11, 14, 16, 25]. Time interval between two consecutive spikes, which is also known as inter-spike-interval (ISI), is the important parameter for temporal coding scheme [1, 14, 25]. Here, ISI distribution becomes a prominent statistical measure to quantify the encoded temporal information. Mathematically, the investigation of ISI distribution becomes the first-passage-time (FPT) problem, *i.e.* the study of time interval distribution of first occurrence of the membrane potential epoch [11, 20, 21, 26]. The analytical study of the FPT problem associated with the neuron model is a difficult task. Explicit expression of solution for the FPT problem is available only for the IF model and in some special cases for the LIF model [20, 21]. To this end, simulation based investigation technique becomes an important tool for obtaining the approximate ISI distribution and for investigating other related neuronal dynamical features.

We investigate the ISI distribution for the MHSN model with four different kinds of input stimuli, namely, constant input, Gaussian distributed input, uniformly distributed input and stochastic input stimuli. We apply the Monte-Carlo simulation technique to yield the approximate solution of the ISI distribution. There are many simulation techniques suggested in literature, we use the following Euler-Maruyama simulation strategy to simulate the three dimensional MHSN model [27, 28].

The total simulation time T Time is divided into n equal subintervals $t_0 = 0 < t_1 < t_2 < \dots < t_n = T$ with size $h = T/n$. In each subinterval, a discrete value of the membrane potential is calculated at the upper time limit. Let V_0 , U_0 and X_0 be the initial values of the variables V , U and X then in subinterval $[t_{i-1}, t_i]$ these variables attains the following value.

$$\begin{aligned} V(i) &= V(i-1) + (X_0(i-1) - U(i-1))h + I(t_i) \\ U(i) &= U(i-1) + a(bV(i-1) - U(i-1))h \\ X_0(i) &= X_0(i-1) + \eta(V(i-1) - X_0(i-1))h \end{aligned} \quad (14)$$

for $i = 1, 2, \dots, n$. Here $I(t_i)$ is the value of applied input stimulus i^{th} discrete time point. Following Izhikevich [3], we use $a = 0.02$, $b = 0.2$, $V_0 = -65$, $U_0 = bV_0 = -13$, $X_0 = 0$, $U_I = 8$ and $V_T = 30$ in our investigation. Fig. 1 illustrates the ISI distribution for the MHSN model with delay parameter $\eta = -0.1$. Here, subfigure (a) depicts the ISI distribution for constant input stimulus of intensity 0.1. Subfigures (b) and (c) are the ISI distribution obtained for standard Gaussian distributed input stimulus and uniformly

distributed input stimulus in range $[0,1]$, respectively. Qualitative behavior of these three ISI distributions is identical. As shown in Fig. 1, they are triangular distributed.

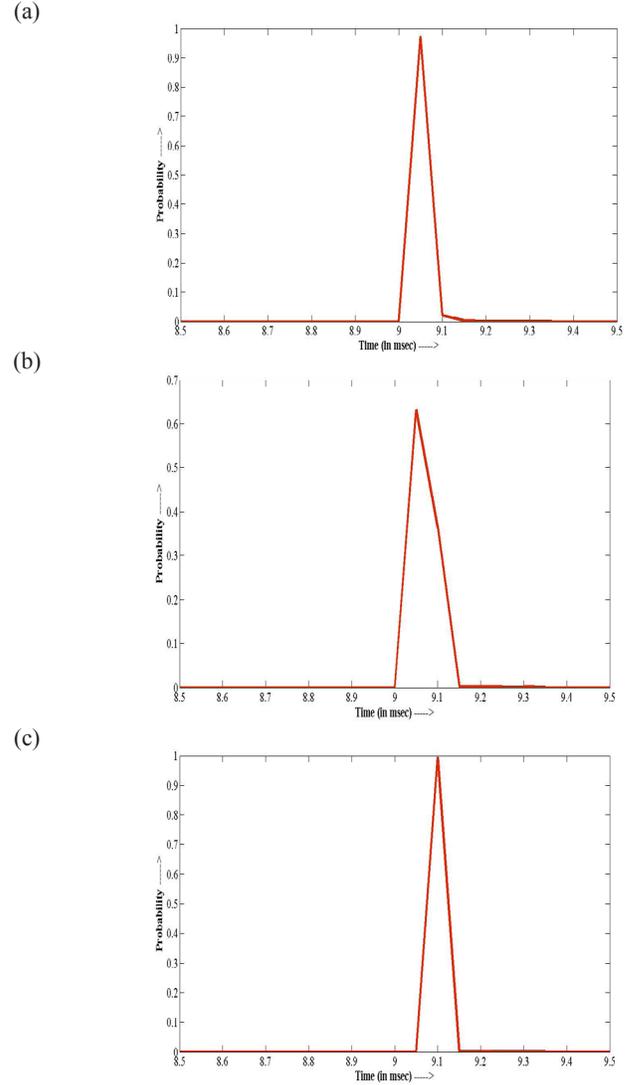


Fig. 1. ISI Distribution for the MHSN Model with (a) Constant Input. (b) Gaussian Distributed Input (c) Uniformly Distributed Input

The ISI distribution for the considered model with stochastic input stimulus having mean μ and noise intensity σ is shown in Fig. 2. It has three subfigures corresponding to three different combination of parameter values η , μ and σ . Subfigure (a) depicts the ISI distribution for $(\eta, \mu, \sigma) = (-0.1, 1, 0.1)$. It has the qualitatively behavior similar to the Fig. 1. In subfigure (b) and (c), we use $(\eta, \mu, \sigma) = (-1, 1, 0.1)$ and $(\eta, \mu, \sigma) = (-1, 0.5, 0.5)$ parameter values, respectively, and notice the similar qualitative behavior but quantitatively changed behavior in ISI distribution patterns. As shown in subfigure (b) and (c), we notice the shift in ISI distribution towards its origin which indicates that the spiking activity of the MHSN model increases due the increase in negative value of delay parameter. This increase in delay parameter works as a memory element so that the membrane potential of the neuron reaches to its firing threshold in comparatively less time.

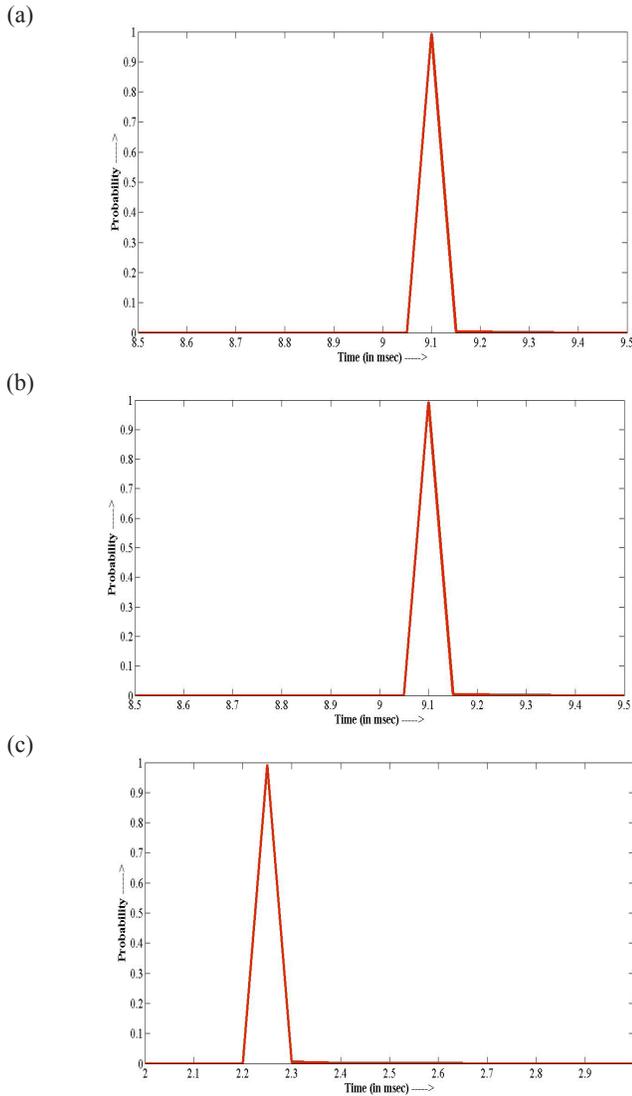


Fig. 2. ISI Distribution for the MHSN Model with stochastic input and (a) $\eta = -0.1$ (b) $\eta = -1$ (c) $\eta = -1$

In order to perform stability in next section, we compute the eigenvalues and steady state solution of the three dimensional MHSN model. Here, we also complete the phase plane analysis of the MHSN model.

III. STABILITY ANALYSIS

Investigation for the evolution of the behavior of state variables is studied under sensitivity analysis [13, 19, 24]. The MHSN model in extended space is a system of three coupled differential equations. It includes two important tasks. In first task, we compute the nullclines and steady state solution for the dynamical system where as second task deals with the phase plane analysis [13, 24]. In phase plane analysis, one analyzes the evolution of temporal behavior of state variables in phase space. In our study, we compute the nullclines, steady state solution, eigenvalues for the dynamical system and perform the phase plane analysis.

The MHSN model given as in Eqns. (11), (12) and (13), is a system of three coupled linear differential equations. Their matrix representation become [13]

$$\dot{Y} = AY + B \quad (15)$$

$$\text{where } Y = (V, U, X_0)^T, \quad A = \begin{pmatrix} 0 & -1 & 1 \\ ab & -1 & 0 \\ \eta & 0 & -\eta \end{pmatrix} \quad \text{and} \\ B = (I, 0, 0)^T.$$

Nullclines and Steady State Solution:

Nullclines are the trajectories in phase space along which the behavior of the state variable changes [13, 24]. Intersection point of these trajectories yields the steady state solution [13]. Nullclines for the dynamical system can be computed by substituting the first derivative

term equal to 0 [24]. Substitution of $\dot{Y} = 0$ in the dynamical system

defined in Eq. (15) results the required nullclines as $X_0 - U + I = 0$

, $bV - U = 0$ and $X_0 - V = 0$. Nullclines for the investigated

dynamical model result a system of three linear simultaneous equations. On solving these simultaneous equations, we get steady state solution

$$(V^s, U^s, X_0^s) = \left(\frac{I}{b-I}, \frac{bI}{b-I}, \frac{I}{b-I} \right), \quad \text{provided } b \neq I.$$

Here, we obtain a single steady state which can be a state or an unstable state which is investigated in next subsection phase plane analysis. Here, we further notice that the model parameter a and delay parameter η don't affect the steady state solution of the model.

Computation for the Eigenvalues:

Let λ be an eigenvalue of the dynamical model (15), and then it can be computed by solving the equation $|A - \lambda I| = 0$ [9, 22]. Its simplification results a cubic polynomial

$$\lambda^3 + (1 + \eta)\lambda^2 + ab\lambda + (ab + 1)\eta = 0 \quad (16)$$

Following the Cardano's method for solving a cubic polynomial,

substitution of $\lambda = Z + \left(-\frac{1 + \eta}{3} \right)$ into Eq. (16) and after some simplification results [29]

$$Z^3 + K_1 Z + K_2 = 0 \quad (17)$$

$$\text{where } K_1 = ab - \frac{(1 + \eta)^2}{3} \quad \text{and} \quad K_2 = \frac{2(1 + \eta)^3 - 3(1 + \eta)ab + 9(ab + 1)}{9}.$$

Following Lal [29], here two cases exist.

Case 1: If $K_1 = 0$ i.e. $\eta = -1 \pm \sqrt{3ab}$

Then eigenvalues for the investigated dynamical system become α ,

$\alpha\omega$ and $\alpha\omega^2$, where $\alpha = (K_2)^{1/3}$ and ω is the cube root of unity.

Case 2: Otherwise, ($K_1 \neq 0$)

Then eigenvalues takes the form as $(P_1 + P_2)$, $(P_1\omega + P_2\omega^2)$

and $(P_1\omega^2 + P_2\omega)$, where $P_1 = -\frac{K_2}{2} + \sqrt{\frac{-4K_1^3 - 27K_2^2}{108}}$ and

$$P_2 = -\frac{K_2}{2} - \sqrt{\frac{-4K_1^3 - 27K_2^2}{108}}.$$

Phase Plane Analysis:

Phase space is a multidimensional space whose coordinates are the system variables [2, 13, 24]. It clearly depicts the dynamical behavior of the state variables [2, 13]. Dynamical system given in Eq.

(15) has three state variables V , U and X_0 , thus we obtain a three dimensional phase space. We simulate this dynamical system same parameter values and simulation strategy as given in Section III.

In Fig. (3), subfigure (a), (b) and (c) show the temporal evolution of state variable with constant input, Gaussian distributed input and uniformly distributed input stimuli, respectively. Here, we observe the limit cycle oscillations generating a spiral structure due to the threshold constraint. This limit cycle oscillation is an important aspect in many biological processes [2].

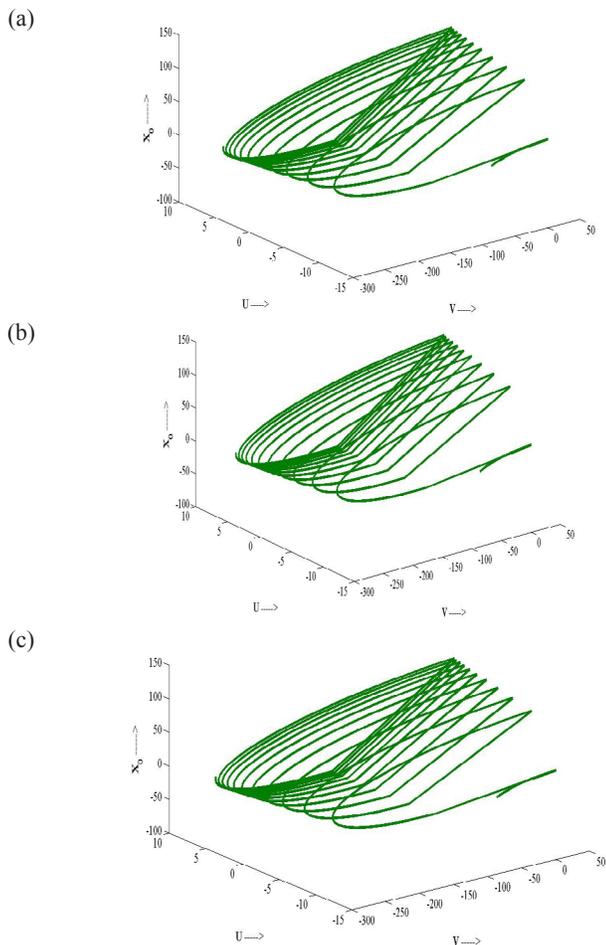


Fig. 3. Phase Plane Analysis for the MHSN Model with
(a) Constant Input (b) Gaussian Distributed Input
(c) Uniformly Distributed Input

Fig. 4 shows the mutual behavior of state variables in the MHSN model with stochastic input stimulus. Here subfigure (a) reflects the evolution of state variables with small magnitude of delay parameter

$\eta = -0.1$ and it qualitatively similar with the trajectories obtained in Fig. 3. We increase the magnitude of η to -1 and maintain the rest parameter values same as taken for Fig. (2). Here, once again, we obtain limit cycle oscillation but time duration of the cycle reduces. This finding suggests that the development of values in state variables occur in quicker time so that the spiking activity of the neuron increases. From Figs. 3 and 4, it is evident that the phase plane dynamics of the model does not alter due to the different kinds of input stimulus; however parameter η reduces the time duration of cycle.

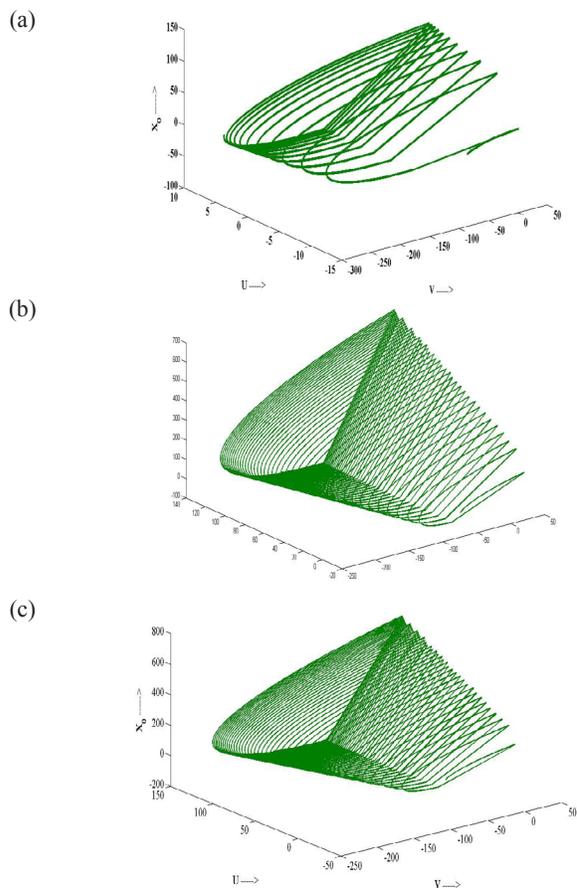


Fig. 4. Phase Plane Analysis for the MHSN Model with stochastic input.

(a) $\eta = -0.1$ (b) $\eta = -1$ (c) $\eta = -1$

IV. CONCLUSION AND FUTURE WORK

The detailed investigation of the MHSN model in term of information processing capability reveals that the spiking activity of the considered neuron model is invariant under the influence of a large fluctuation of applied input stimulus. The state variable X_0 of the model in extended space along with the delay parameter η curtails the variation in spiking activity. However, increase in the negative value of η increases the spiking of the neuron. This delay parameter works as a memory element and helps the membrane potential to reach its firing value in quicker time so that the ISI reduces and the ISI distribution gets scaled with minus values as depicted in Fig. 2. We notice a single unstable steady state solution of the model during stability analysis

which is independent of delay parameter. Choudhary *et. al.* [30] has shown that “*the distributed delay has no effect on stationary state membrane potential distribution of a LIF neuron*”. In addition, we say that *the distributed delay has no effect on steady state solution of the threshold based linear neuron models*.

Being the extension of Izhikevich neuron model [3], the MHSN model has the inherited property related to the implementation of after spiking reset condition at hardware level. Invariant spiking activity of the MHSN model reveals that the model is capable to handle the threshold variability and other noisy parameters like temperature increment in the chip. Lim *et. al.* [9] has implemented the neuristor-based leaky integrate-and-fire neuron model with aforementioned two critical issues to form an artificial neural network at hardware level. As the MHSN model is free from above stated prime issues, we recommend that *the MHSN model should be a better choice for chip level implementation*.

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