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Arithmetical Certainties: A Few Exceptions Among Countless Knowledge-Statements.

Certezas aritméticas: algunas excepciones entre innumerables enunciados de conocimiento.

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Resumen
En este artículo discrepo de Kusch (2016) en tres cuestiones relacionadas con las expresiones de certezas aritméticas –en el sentido de Wittgenstein– y los usos regulares de las expresiones aritméticas. Específicamente, explico por qué los cálculos no se convierten en certezas por el hecho de haber sido probados; Argumento que los cálculos probados constituyen enunciados de conocimiento; y, por último, pero no menos importante, concluyo de esto que tales cálculos probados son decibles, mientras que las certezas aritméticas son inefables o indecibles.
Palabras clave: Certezas, Conocimiento; Aritmética; Wittgenstein; Kusch.

Abstract
In this paper I disagree with Kusch (2016) on three issues concerning expressions of arithmetical certainties – in Wittgenstein's sense – and regular uses of arithmetical expressions. Specifically, I explain why calculations do not turn into certainties by the fact that they have been proved; I argue that proved calculations constitute knowledge-statements; and, last but not least, I conclude from this that such proved calculations are sayable, whilst arithmetical certainties are ineffable or unsayable.
Keywords: Certainty; Knowledge; Arithmetic; Wittgenstein; Kusch

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1 · Introduction

One of the most characteristic traits of mathematics for traditional epistemologists is the lack of choice within this realm. As is the case with logic, mathematical truths are absolutely necessary, so that "2+2" inexorably equals "4". It cannot be otherwise. This inexorability attracted the attention of Ludwig Wittgenstein, who remarked in his posthumous work *On Certainty* that the absolute certainty of arithmetical propositions like the multiplication tables is also characteristic of what appear to be empirical propositions such as "This is my hand" (OC §§448, 653)[2], for they are all used within their respective language-games without any doubt or hesitation (*cf*. OC §447)[3]. McGinn (1989) and Moyal-Sharrock (2004) provided a non-epistemic rendering of Wittgenstein's views on certainties and mathematical sentences according to which none of them are propositions because they are true in an empty sense; they can never be epistemically justified; they are in all senses prior to empirical knowledge; they cannot be uttered in a meaningful way; and they constitute grammatical rules. These five tenets were challenged by the insightful epistemic view developed by Kusch (2016) following Pritchard (2007), Williams (2004a; 2004b; 2007) and Wright (2004). As I will not discuss whether certainties should be regarded as propositions, in this paper I do not fully enter this debate; however, I will address three issues closely related to it. To begin with, and with the aim of providing a foothold for my subsequent reflection, I will show that we cannot be certain of the entirety of calculations. Later I will disagree with Kusch (2016) on three issues. Specifically, I will explain why calculations do not turn into certainties by the fact that they have been proved; I will argue that proved calculations constitute knowledge-statements; and, last but not least, I will conclude from this that such proved calculations are sayable, whilst arithmetical certainties are ineffable or unsayable. By this I do not mean that my contribution corresponds to Wittgenstein's definite view, for, as Kusch rightly pointed out, Wittgen-

2 However, Wittgenstein also considered some objections to this in OC §654.
3 As Wittgenstein had previously indicated in his Philosophical Investigations, a "language-game" is "the whole, consisting of language and the actions into which it is woven" (PI §7). Examples of language-games include, among many others, "Giving orders, and obeying them", "Reporting an event", etc. (PI §23).

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stein's remarks on mathematics "are full of expressions of hesitation and doubt" (122). Yet I think that my contribution constitutes an approach to the characterization of arithmetical certainties which may be useful to further develop the debate between the non-epistemic rendering of certainty and the epistemic one.

2 · Characteristics of arithmetical certainties

Before analyzing the main features of arithmetical certainties, it is appropriate to present Wittgenstein's notion of "certainty". To this end, I will start by clarifying the categorial distinction that Wittgenstein established between knowledge and certainty (*cf*. OC §308). Specifically, he regarded knowledge as justified true belief: thus, if someone says "I know", she must be able to demonstrate the truth by dispelling a doubt, so that her grounds must be surer than the assertion of what she believes (*cf*. OC §243). Thus, the assurance "I know" should only be given to someone who still has not become aware of the grounds of such knowledge-statement (OC §431). Conversely, certainty is ungrounded: though we might provide many grounds for supporting a certainty, none would be "as certain as the very thing they were supposed to be grounds for" (OC §307). Certainty can therefore be regarded as a spontaneous "attitude" (OC §404) which we daily and implicitly show in whatever we say and do (*cf*. OC §431) without the aim of removing any doubt. Countless certainties can thus be simultaneously shown without generating inconsistency, for they are far from being independent of each other: in fact, our certainties make up a "world-picture" (OC §§93-95) which constitutes the "background" against which we distinguish between true and false (OC §94).

Most importantly, the possibility of making a mistake is "logically excluded" when it comes to certainty (OC §194). Wittgenstein was not referring here to the classical and context-independent conception of logic which is valid in all possible worlds (*cf*. OC §628), but to a peculiar logic that emanates from the rules that make up our language-games (*cf*. Author 2021). This indicates that Wittgenstein did not aim to provide a merely theoretical and decontextualized distinction between knowledge and certainty.

Indeed, and following the priority given to descriptions characteristic of his *Philosophical Investigations* – where Wittgenstein intended to dissolve philosophical problems or conceptual confusions by describing language-games (*cf*. PI §109) – his interest in descriptions remained in *On Certainty*. For he did not aim here to state how people *should* behave; instead, he wanted to show how they *do* behave (*cf*. OC §284). That is why Wittgenstein does not indicate which errors should be excluded from our language-games; instead, he simply reveals that there are seeming mistakes that we do not accept as such within our language-games, which can be considered as a spontaneous expression of our very lives (*cf*. OC §559)[4]. While the mistake someone may make when uttering a knowledge-statement can only be made within a language-game whose rules indicate what is considered as an error and how it could be discovered (*cf*. OC §§196, 301), a seeming mistake concerning certainty could not be regarded as a mere – albeit huge – error but as an anomaly or irregularity that "happens as an exception" because there is no place for it in our language-games (OC §647). Such an anomaly would open up what Author (2013 137) called a "grammatical gap" between the concerned individual and her community of origin, for it would not be possible to discern what she "would still allow to be counted as evidence and what not" (OC §231).

After making this brief introduction to Wittgenstein's conception of "certainty", I will now focus on arithmetical certainties. In order to shed light on this expression, I will begin by clarifying whether all calculations constitute arithmetical certainties. In this vein, it seems reasonable to assume that there exists an immutable mathematical realm in which all arithmetic operations are contained. According to Platonism, "2+2=4" constitutes an eternal truth inasmuch as it refers to a fact of a mathematical reality that is independent of human existence, as a result of which the mathematician must increasingly discover the mathematical relationships and properties that such reality contains. Wittgenstein, however, was far from being a Platonist (*cf*. Kober): instead, at least between 1929 and 1944, he seemed to be much closer to formalism (*cf*. Rodych). After all, Wittgenstein

4 With very few exceptions – like language-games characteristic of military communication – our language-games arise and develop spontaneously, without anyone creating or modifying them at will.

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claimed that "the mathematician is not a discoverer: he is an inventor" (RFM II §2), for, in his opinion, "we can't describe mathematics, we can only do it" (PR §159). Wittgenstein would therefore not regard arithmetic as a pre-determined and immutable structure that the mathematician can discover and describe one part at a time; from his standpoint, mathematics merely constitutes a practice or language-game that has become established due to the concordance and consistency with which it has been used[5]. In view of the above, it seems strange that Wittgenstein claims in *On Certainty* that "[t]he propositions of mathematics might be said to be fossilized" (OC §657), and that they have "as it were officially, been given the stamp of incontestability" (OC §655). Seemingly, he refers to all arithmetical operations, as though all of them were certainties. It should be noted, however, that the examples of such operations provided by Wittgenstein are only two, i.e. "12x12=144" (OC §§43, 651, 653-654) and "the multiplication tables" (OC §658). The way I see it, it is highly significant that these examples do not concern complex calculations whose results must be estimated in order to be known, but a small group of arithmetical certainties shared by the large majority of educated adults: for, regarding these elemental calculations, it can be asserted without any hint of doubt that they have really been given the stamp of incontestability[6].

Of course, the shortage of textual evidence does not ensure that Wittgenstein only considered as arithmetical certainties multiplication tables and basic calculations of the kind of "12x12=144"[7]. But I should like to briefly

5 Wittgenstein (OC §47) recommended focusing on our customary practice of calculating, thereby leaving aside "transcendent certainty". As Weiberg (2020) pointed out, this transcendent certainty could be understood as "a rule from which it follows that there can't have been a miscalculation here" (OC §44), but we might also "go wrong in applying it" (OC §26).

6 According to Hand, there are two ways of imparting a belief to a student, that is, "by proving it to her or by indoctrinating her" (549). In this vein, Siegel warned that teachers are always obliged to offer reasons for all beliefs and practices: from this standpoint, a fully rational education should transform ungrounded certainty in grounded knowledge. However, "grounded knowledge will be unfeasible unless we previously rely on countless ungrounded certainties" (Author 2019a 403).

7 It cannot be exactly indicated which calculations constitute arithmetical certainties, as that will depend, among other factors, on age, education, personal capacity for arithmetic, and, as I will show in section 2.3, personal experiences. As a guideline, it can analyze what the consequences would be if all calculations were certainties. In such a case, arithmetical certainties would owe their status to their belonging to an immutable order that is completely independent from people's lives, which would result in at least two peculiar scenarios. On the one hand, it might then seem that there can be arithmetical certainties of which no one is certain or even aware. Yet, as we are considering Wittgenstein's notion of "certainty", arithmetical certainties, like any other certainty, must be shared – that is, they should be shown or reflected in whatever is said and done by people who play arithmetical language-games: for, as indicated above, the logical exclusion of mistakes, which constitutes the main feature of certainties in Wittgenstein's sense, emanates from practices whose daily repetition by many people has given rise to consolidated language-games. On the other hand, it could then also be questioned whether such certainties are shared by all of us. But the answer is negative because we all, including the most notorious mathematicians, are only informed of a negligible number of calculations: it is therefore hard to understand how we could be certain of those calculations whose results we do not know until we compute them. Additionally, if someone were certain of the results of all possible calculations, she could not make mistakes or have any doubt about any of them, because such seeming mistakes or doubts would then be anomalies for which there would be no room in our arithmetical language-game. Nevertheless, this language-game admits errors and doubts concerning complex calculations – i.e. those calculations that are not established as certainties – but not regarding arithmetical certainties. At this point, it could still be argued that many people are certain that all possible calculations make up a perfectly harmonious system in which their results are unquestionable. Yet, though such certainty is widely shared, it should not be confused with the certainty concerning specific calculations. For the fact that someone is certain of the existence of an arithmetical system invariable throughout eternity does not entail that she will thereby be certain of the result of complex calculations: after all, she will not even know their results before she calculates them.

be said that an educated adult will usually be certain of the results of at least additions, subtractions, multiplications and exact divisions with numbers lower than 11.

Once explained why we cannot be certain of all calculations, I will now clarify why I disagree with Kusch on three relevant issues concerning arithmetical certainties.

2 · 1 · Are calculations certain by the fact of their being proved?

I have just shown that arithmetical certainties do not include the entirety of calculations, but only a small group of them. In the present section I will go one step further by specifying which arithmetical operations are excluded from being certainties. To this end, I should like to start by noting that, according to Kusch, arithmetical certainties are "mathematical sentences that have been immunized by mathematical proofs", in addition to which "they are certainties for us all" (128). Before explaining why I disagree with this, I should like to emphasize that I am referring to extremely basic arithmetical proofs, such as proving subtractions – for instance, a–b =c through c+b=a – or divisions – a:b=c through b·c=a – to give only two examples. Specifically, the distinction that Wittgenstein established between "experiment" and "calculation" in his *Remarks on the Foundations of Mathematics* may lead us to consider – in the context of this paper – "experiment" as the initial calculation we make in order to know an answer or result, while the paradigmatic "calculation" constitutes the further proof with which we can optionally check the result previously obtained:

> **If we take a pile of 100 marbles and separate them into 10 groups of 10, that is an "experiment" in the sense of counting how many marbles are there (RFM I §36ff.). When we begin, we do not know the answer. But if we now film the procedure, regarding it as a paradigm of what can always be done, as a necessary feature of there being 100 marbles, then we regard the procedure as a paradigmatic "calculation" or proof: we have "unfolded" the implicit properties in the collection. We "see the necessity" in the procedure: the way of regarding typical of proof. (Floyd 57-58)**

Having said this, I will now explain why I disagree with Kusch when he claims that arithmetical certainties are "mathematical sentences that have been immunized by mathematical proofs", in addition to which "they are certainties for us all" (128). Let us thus suppose that a number of mathematicians have converged in finding out the result of a complex calculation at the same time and independently from each other. Yet even though they agree that the proof is correct – and that the calculation has thereby received the stamp of incontestability – this does not mean that such calculation will automatically turn into a certainty. Regardless of whether the result has been proved, one should exert oneself to remember it: but, of course, certainty is not something that requires an effort for being remembered. Hence, in this case there would simply be a proved calculation that some people *want* to treat as a certainty. Yet Author (2016) has provided four arguments as to why certainties cannot be acquired at will. Now I will resort to those arguments in order to explain why a proved calculation cannot be converted into an arithmetical certainty solely on the basis of one's own will.

Firstly, certainty cannot be acquired through reasoning. Wittgenstein wrote:

> **And now if I were to say "It is my unshakeable conviction that etc.", this means in the present case too that I have not consciously arrived at the conviction by following a particular line of thought, but that it is anchored in all my questions and answers, so anchored that I cannot touch it. (OC §103)**

We cannot therefore touch or reach an arithmetical certainty through a conscious process such as a mathematical proof, for Wittgenstein regards certainties as ungrounded (*cf.* OC §§250, 307). The impossibility of reaching a specific certainty through reasons or proofs also concerns the world-picture, that is, the set of all our certainties: for the world-picture is not something we decide to adopt after reflecting thereon, as it constitutes our "inherited background" (OC §94). If a certainty could be acquired by the mere fact of finding out a reason or proof that supports it, then it would be an explicit learning; however, Wittgenstein warns that he does not "explicitly learn" such certainties: instead, he can only "discover them *subsequently* like the axis around which a body rotates" (OC §152, emphasis by the author). At this point it could be argued that the acquisition of a certainty is not completely external to the individual, for she can foster or avoid situations that facilitate its subsequent assimilation. By way of example, it will be more likely that someone ends up assimilating as certain a complex calculation like "7,765x3,267=25,368,255" if she repeatedly uses it in a variety of contexts; but this does not entail that the acquisition of such certainty depends on the individual's will. Instead, and strictly speaking, it will be something that *happens* to her.

Secondly, certainties are assimilated regardless of whether or not we are in a specific mental state. According to Wittgenstein, neither certainty nor knowledge are mental states like, so to say, "being sure" (OC §308). The main feature of certainty is the logical exclusion of mistakes thereon: thus, keeping in mind that such exclusion stems from our language-games, we should focus on them and not on our mental states in order to conclude whether there is room for a specific doubt or mistake (*cf.* OC §§601, 196). This is particularly clear in the case of arithmetic, for we do not check whether someone has calculated rightly by focusing on her mental state, but by seeing how she has used arithmetical rules (*cf.* OC §§30, 38). A certainty cannot therefore be attained by the mere fact of reaching a specific mental state.

Thirdly, an arithmetical certainty cannot be acquired even when one is willing to accept the certainty or world-picture proposed by a persuader. Indeed, when two individuals or groups meet each other without sharing the same world-picture, none of them can be convinced about particular issues by being given grounds: in such cases, one party may attempt to *persuade* the other by giving one's own world-picture (*cf.* OC §262). Churchill remarked that one "decides" whether she becomes persuaded (46); however, such a decision would simply constitute a mental state which can be regarded as "a mere willingness towards the conversion" (Author 2016 578). Yet the assimilation of a certainty does not depend on any mental state, so that persuasion is irrelevant to this end.

Fourthly, a certainty cannot be attained by acting *as if* one had it, as an act of will shall then be necessary not to forget that one should not stop acting in tune with that certainty, or to clarify how to act according to it in particular cases. From this it follows that the concerned individual would be considering the possibility of making a mistake when manifesting a certainty; but Wittgenstein (*cf.* OC §§490, 494) remarked that there is no judgement of which one could be certain if she had the slightest doubt about a certainty. Thus, the decision to acquire an arithmetical certainty by acting as if one had really assimilated it would result in a simulation which would be incompatible with its genuine and spontaneous manifestation.

2 · 2 · What, then, are calculations once proved?

I have just argued that proof does not suffice to turn a calculation into a certainty because the proof will at most generate a mere willingness that is not enough on its own to bring about the acquisition of such certainty. In saying this I am by no means implying that arithmetical proof is completely insignificant: instead, I merely want to say that when a calculation is proved, it does not thereby turn into a certainty, as then it gives rise to grounded *knowledge*. Indeed, the proof – or reason – is in this case surer than the result – or assertion – for it is accepted that the latter will vary depending on the former (*cf*. OC §243). Thus, if we initially check that "49x37=1813", we may conclude that this calculation is right; but if we intended later to prove it – by calculating 1813/37 – and then realized that the initial calculation was wrong, we would put the result into question – unlike for certainties – until new and definitive proof allows us to acknowledge that we now know that "49x37=1823". In this vein, Wittgenstein claims: "When does one say, I know that … x … = …? When one has checked the calculation" (OC §50). This use of the expression "I know that … x … = …" is encouraged from a young age: to give one example, Ryan and Williams have distinguished four developmental categories of mathematical errors, in addition to which they provided dialogical category-related strategies in order to facilitate that children understand and thus correct the errors that they made even when claiming to know the result of a specific calculation[8]. Teachers therefore handle foreseeable error patterns as well as their plausible explanations, but arithmetical mistakes may also be made by adults who had previously claimed to know the result of the corresponding calculation.

Thus, as Wittgenstein suggested, "out of a host of calculations certain ones might be designated as reliable once for all, others as not yet fixed"

8 It should be noted that the fact of saying "I know" does not exclude the possibility of mistake (*cf.* OC §§12, 21, 127), which leads Wittgenstein to suggest that the expression "I know" is replaced by "I believe I know" (OC §366).

(OC §48). Some calculations – like the multiplication tables – end up being assimilated as certain, but the vast majority of them do not become fixed. This is reflected by our arithmetical language-games. For instance, no one is asked about a calculation as basic as "3+3=…" unless it is a child who is still learning to calculate. Moreover, when we ask an educated adult about a calculation and she gives us a result, we do not further ask her *how* she knows it, for the answer is entirely predictable: she knows it *because* she has calculated it by herself, or *because* she has obtained that information from some source (*cf*. OC §550). The conjunction "because" leads us here to a proof that provides the epistemic distance which distinguishes knowledge from certainty. The difference between both categories is also clear when the focus is placed on what we consider either as anomaly or as mistake in calculating. By way of example, if a very young pupil made the calculation "5+3=2", her peers would surely contemplate it as an explicable mistake, as they might argue that the pupil did not realize that she had to add instead of subtracting. Now, if these learners were told that someone who can add fluently and does not seek to mislead them made the calculation "5+3=626,837,465", the result would appear so strange that they could suspect that it should not be considered as a mistake, for it seems to be "a huge number randomly written instead of an attempt to solve the calculation" (Author 2019b 1035).

2 · 3 · Are arithmetical certainties sayable?

We have just seen that proved calculations do not constitute certainties, but mere knowledge. In this section I will show that, precisely because proved calculations can be expressed through knowledge-statements, such calculations are sayable – i.e. they can be meaningfully uttered because there is room for them in our language-games (*cf*. OC §348) – whereas arithmetical certainties are ineffable. Indeed, Wittgenstein (*cf*. OC §35) uses the term "nonsense" to refer to those strings of words that "have no use within a language-game – that is, there is no *recognized context* or *circumstances* in which the expression functions" (Moyal-Sharrock 94). Of course, it may happen that a context is found in which such sentence no longer strikes us as meaningless (*cf*. OC §469); but if no context can be found in which it gains sense, then it will remain nonsensical (*cf*. OC §468). This kind of sen-

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tences can be uttered or verbalized, but not within a language-game: hence, they have no sense. The same applies to certainties, whose role is supportive of the language-game but "it is not work *within* the game" (Moyal-Sharrock 45). According to Kusch, however, it makes good sense to report not only mathematical certainties to others, but also basic scientific doctrines that "often turn into proverbs or platitudes (*cf*. Shapin 2001)" as well as "religious certainties" which "are typically repeated again and in again in prayers" (137). Before analyzing in greater detail what Kusch says about the ineffability of mathematical certainties, I should like to comment on the other two examples he refers to, i.e. scientific laws that often turn into proverbs, and religious certainties which are repeated in prayers.

Goody remarked that scholars usually begin to analyze proverbs by making lists of them, thereby testing them for a universal truth value, "whereas their applicability had been essentially contextual" (125-126). In this vein, Shapin used the term "proverbial economy" to consider proverbs not "as naked propositions-on-a-printed-page", but "as speeches in situations of use" (745). Proverbs are therefore not easy to store, choose and apply accurately. By way of example, if someone suddenly claimed "There is a remedy for everything except death" or "Every door may be shut but death's door", without making it clear why, such assertions would be as strange as the literal formulation of a certainty; yet, in very specific situations, "some generations have found in these sayings a way of not only expressing their anguish, but also of facilitating their resignation in the face of such an inevitable destiny" (Author 2015 83). Of course, proverbs have also been used in other contexts. Regarding canonical scientific laws, if a doctoral candidate made a mistake in her research project by not taking into account some biological process, her thesis supervisor might tell her: "Nature doesn't make leaps". Despite appearances, this does not constitute the expression of a basic certainty within scientific research, for what the supervisor meant here was something like: "You must be patient and focus on the whole chain of events you have to investigate". In other words, the proverb was not uttered with the aim of literally expressing a certainty, but simply for the purpose of making the doctoral candidate calm down enough to think more clearly and thus to become aware of the steps she had forgotten. In fact, if the candidate did not understand the proverb in this way and took it literally, as a piece of information intended for her because she allegedly ignored it, she would become astonished until she found a context – like the one I have mentioned – in which such statement gains sense (*cf*. OC §469). This example illustrates that, as Shapin (761) pointed out, proverbial economies can also be applied to the real-world scientific practice; but, I would add, without thereby constituting expressions of certainties.

Regarding religious certainties that, according to Kusch, are repeated time and again in prayers, it should be noted that religious believers often lack such certainties: for doubt is a basic constituent of faith (Verbin), as a result of which religious believers may be concerned with disproof, to the extent that they often change their minds when they cannot account for evidence against their faith (Boudry and Coyne). Hence, the believer whose faith is related to doubt can pray to strengthen her faith; conversely, whoever shares religious certainties may resort to prayers, among other things, to ask God for something or to ease her own conscience but not to bolster her certainty: for certainty neither needs to nor can be strengthened, to the extent that the very attempt to bolster it would be incomprehensible to the concerned individual. A religious certainty would therefore not be expressed in these cases by the fact of uttering sentences of the kind of "Our Father, Who art in heaven". However, Author noted that religious certainties differ from other certainties because some world-pictures contain the former while others exclude them, so that a religious certainty might be claimed, to give some examples, "in order to posit a distinction between oneself and other people; in order to locate moments in which one's world-picture was different; and in order to pretend that one shares a religious certainty" (2020 662). Thus, religious certainties seem to have been expressed in these exceptional cases; yet, strictly speaking, the aim of such statements would simply be to highlight a – sometimes feigned – difference between one's own world picture and other ones, or one's own at another time.

After having referred to the ineffability of certainties by analyzing the cases of scientific laws that often turn into proverbs and religious certainties which are repeated in prayers, now I want to show that, although Kusch claims that it makes good sense to report mathematical certainties to others, such reports only make sense when they constitute knowledge-statements, i.e. when they are aimed at dispelling a doubt and therefore have a role with-

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in our language-games. I will draw upon a simple example to illustrate this point. Let us suppose that a crowded bakery sells every day a huge amount of rolls that cost 19 cents each. Customers may know the price they have to pay for a specific amount of rolls *because* they have calculated it by themselves, *because* they have watched the price at the cash register, or simply *because* the baker has informed them thereof. All these grounds are surer than the result indicated in the corresponding knowledge-statement, for they can be used to correct mistakes. Meanwhile, the prices of rolls between one and a reasonable amount would have been established as certainties for the bakers. Originally, these prices would have also been known by them, but thanks to a number of practices that were repeated time and again – such as making the calculations almost steadily, indicating them as prices without customers calling them into question, and realizing that income is balanced with the number of rolls sold every day, among others – at some time which cannot easily be specified bakers would end up assimilating as certain diverse arithmetic operations by multiples of 19 cents. Admittedly, reasons or proofs would have been considered by bakers while they used calculations as knowledge-statements subject to the possibility of mistake; yet once such calculations became certainties, reasons would no longer work as such because they would have ceased to be surer than the assertion or result they supported. Proof of this is that none of these bakers would consider as a reasonable – or even "legitimate" (OC §315) – doubt the fact that a customer gave a reason to call into question one of the prices that bakers regarded as certain. In this vein, if a baker said to a customer that 3 rolls costed 56 cents, this statement would not be regarded by bakers as a mistake but as a slip of the tongue, which would confirm that they are certain of "19x3=57" (*cf*. Author 2012)[9].

3 · Conclusion

In this paper I have argued that arithmetical proof constitutes the ground that supports a knowledge-statement which, in turn, can become a certainty

9 Although the baker and her workmates explicitly claimed that she had made a "mistake" when saying that 3 rolls cost 56 cents, they would all take for granted that she was certain of the right result – without admitting any hint of doubt – even when she made the seeming mistake.

when the ground is no longer regarded as a proof to dispel a doubt or to detect possible errors. Indeed, children start by knowing that "2+2=4" or "10−5=5" *because* they have calculated it by themselves or *because* their teacher has told them so; but there comes a time when, if someone intended to convince them that they had been wrong until then in making such calculations, they would no longer take those claims seriously. Additionally, the acquisition of a certainty does not occur automatically by the mere fact that a compelling ground is – or rather, seems to be – supporting it. Thus, proofs or reasons support knowledge-statements, while certainties are ungrounded; but, most importantly, the acquisition and the development of certainties are to a large extent unforeseeable and hard to trace. The latter issues may remain out of focus when the emphasis lies on whether arithmetical certainties are propositions; but, following Wittgenstein (*cf*. PI §109; OC §284), I have given priority to the description of current uses of language. In this manner, I have revealed relevant aspects concerning the origin, development and expression of arithmetical certainties, thus emphasizing that they are certainties not by the fact of their having previously been proved, but by the way in which they are reflected in whatever we daily say and do, as is the case with the rest of certainties.

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